Debt, Inflation, And Government Reputation*

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Abstract

This paper develops a theoretical framework to explain the correlation between public debt and inflation through different episodes, focusing on the role of government reputation (defined as the public's belief in the government's commitment to low inflation) in shaping inflation expectations. Many countries, particularly in Latin America, have experienced periods of high inflation driven by elevated public debt and fiscal deficits. While independent monetary authorities and inflation targeting have weakened the historical link between debt and inflation, concerns persist that high debt could still trigger inflation. I propose a dynamic game model with incomplete information where private agents (wage setters) and a consolidated government interact over time. The government can be either prudent, prioritizing low inflation, or imprudent, favoring short-term output and debt gains through higher inflation. Wage setters form inflation expectations based on the government's debt trajectory and its perceived reputation. The model implies a monotonic relationship between inflation and reputation, in the sense that higher government reputation implies lower inflation. In addition, as government reputation increases, the incidence of the current debt state on inflation is reduced. Hence, when reputation is strong, a government can sustain low inflation even with high debt. I calibrate the model using data from four emerging markets (Mexico, Colombia, Guatemala, and Thailand), illustrating how government reputation influences inflation dynamics. The findings underscore the importance of maintaining low inflation as debt rises to build and preserve government credibility, while also providing insights into the periods of high correlation between debt and inflation observed in these economies.

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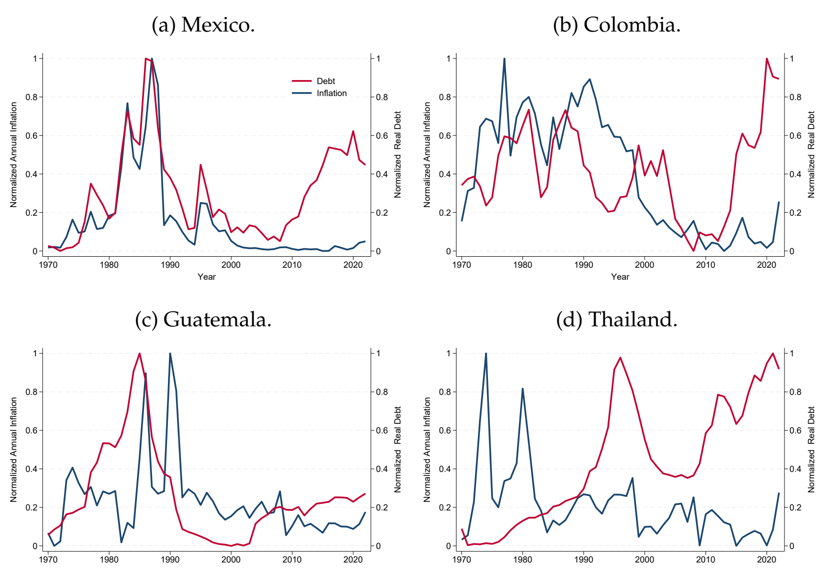
1 Introduction

During the 20th century, many countries experienced high levels of inflation, or even hyperinflation. Several papers in the literature suggest that most of these episodes were a consequence of elevated debt and fiscal deficits which eventually led to a spiral between expected and observed inflation. For example, Sargent et al. (2009) analyze the case of several Latin American countries that had high inflation during the 80's, mainly caused by high debt levels financed through money creation, which destabilized inflation expectations. These authors argue that even when fundamentals such as debt and deficit were under control, it took several years or even decades to anchor expectations back to a low level. In order to achieve this, the government of these countries had to implement credible reforms that would convince the public that the government was committed to low inflation (Fischer (1995), Sims (2016)).

Figure I displays inflation and debt data for different emerging market economies. These countries experienced periods of high inflation alongside high public debt, as well as times when rising debt did not trigger significant inflation. This structural shift partly reflects the establishment of independent monetary authorities and the adoption of inflation targeting. However, recent literature (e.g., Kocherlakota (2012), Bassetto and Miller (2022)) suggest that high debt could still lead to higher inflation, even with independent monetary policy, due to fears of Central Bank bailouts or regime shifts. This raises important questions about the persistent correlation between debt and inflation, even under Central Bank autonomy.

Figure I: PUBLIC DEBT AND INFLATION 1970 - 2022.



NOTES: These panels display inflation (blue) and debt (red) time series for each country, normalized to be in the [0,1] interval. For more details on the data, consult Section 6.1.

This paper develops a theory to explain why debt and inflation are highly correlated in some periods while appearing disconnected in others. The key lies in the private sector's belief in the government's commitment to maintain low inflation, which I refer to as *government reputation*. When the public perceives a weak commitment, the correlation between debt and inflation is strong: agents anticipate higher inflation, and thus, when debt rises, the government has an incentive to actually deliver what is expected and erode debt through inflation, creating a positive and pronounced link between the two variables. Conversely, as confidence in the government's commitment to low inflation grows, this correlation weakens. In such cases, maintaining low inflation helps the government bolster its reputation, since in this scenario agents expect low inflation.

I develop a reputation model, i.e. an incomplete information dynamic game with imperfect monitoring in which private agents (wage setters), who care about setting their wage in order to have a constant real wage (taking as given the price level), interact with a consolidated government, who cares about the output gap, inflation, as well as the debt it generates. The government can be one of two types: an imprudent government who cares less about the evolution of debt and inflation, tending to generate high inflation and debt in order to boost output (type ξ^I); or a more prudent government that has a stronger commitment to low inflation (type ξ^P). The government's type is private information, and wage setters can only observe a noisy signal of government's actions. In this framework, I understand government reputation as the probability that wage setters assign to be facing the government with higher commitment to low inflation, upon observing the history of previous play. This repeated interaction constitutes a dynamic game since agents take into account two state variables (current debt and government reputation), which influences both their payoffs and actions.

In the stage game that agents play each period, wage setters decide their wage and the government simultaneously chooses the money supply (which in turn pins down inflation), deficit, and debt. The government's objectives include boosting output, which can be achieved through higher inflation or (primary) fiscal deficit, while also maintaining a controlled level of inflation and debt. Since from the point of view of the government wages are fixed (they are decided by other player), it has an incentive to "inflate away" the wage of workers in order to boost output in the short-run. This is what Kydland and Prescott (1977) refer to as the time inconsistency problem of monetary policy. In my setting, this incentive discrepancy is worsened whenever debt is high, since the government, aiming to prevent a debt explosion, will avoid generating high deficits, having to rely on monetary policy to boost output and reduce debt. Agents understand this and, in turn, demand higher wages whenever they observe that they are in an elevated debt state.

In my framework there is a separating equilibrium behavior of the government of type ξ^P, in the sense that ξ^P chooses different actions than ξ^I no matter the state the economy is in. However, the degree of separation is heavily influenced by both debt and reputation levels. A prudent government prefers to have low inflation today, however, when deciding what to choose optimally it also takes into account the impact of its decisions on both the evolution of future debt and of its reputation. Generally, the prudent government faces a trade-off between higher future debt and higher future reputation. To build reputation, the prudent government needs to choose an inflation level that distinguishes it from the imprudent type, typically requiring lower inflation. However, choosing lower inflation leads to higher debt (since low inflation implies lower seigniorage and a higher real interest rate on previous debt), which is detrimental to the government's long-term objectives. If the current government's reputation is low (i.e., agents believe they are likely dealing with an imprudent government), the cost of choosing an inflation level low enough to rebuild reputation (which would significantly increase debt) may outweigh the benefits. In such cases, it is optimal for the government to choose similar inflation and deficit levels to what an imprudent government would choose, as this strategy reduces debt without altering the current reputation level. Thus, the possibility that agents might be facing an imprudent government forces the prudent government to behave more imprudently when its reputation is low, generating a sequence of debt and inflation with high correlation. Conversely, when the government's reputation is high, the prudent government can afford to choose a lower inflation rate to further enhance its reputation, even if this results in higher debt than what an imprudent government would generate. Under this scenario, a prudent government would produce inflation time series that is less correlated with debt.

In the model, government reputation is also reflected in wage setters' inflation expectations. When the government's reputation is low, inflation expectations rise because agents anticipate that they are dealing with an imprudent government likely to generate higher inflation rates. According to the model, whenever we observe both inflation expectations and debt increasing (and thus a high positive correlation) it suggests that the government is "losing reputation," meaning agents believe they are facing an imprudent government. Conversely, when the correlation between expectations and debt is weak, it indicates high government reputation. This is a testable prediction of the model that I take to the data to see if the agents of an economy believe they are facing a government committed to low inflation or not.

Although the primary contribution of this paper is theoretical, to further understand the model dynamics and implications, I calibrate it in order to account for the inflation and debt time series of four emerging market economies (Mexico, Colombia, Guatemala, and Thailand) during 1970-2022. The model generates a series prediction for government reputation, inflation expectations, fiscal deficit, and output gap that are consistent with the framework and the observed inflation and debt data. Generally, the model underscores the importance for a government to control inflation as debt increases, as this helps to accumulate reputation. Episodes where debt raises alongside inflation tend to erode government reputation, as seen in Mexico (1982-1992; 2015-2022), Guatemala (1988-1998), and Colombia (1980-2010). The model estimates that these countries have managed to build high reputation over the past twenty years, as inflation has stabilized and remained low despite increasing debt. However, recent challenges have arisen, particularly during 2020-2022, where debt increased alongside a spike in inflation, leading to a slight decrease in government reputation.

Among the countries analyzed, Thailand stands out as the only case where high inflation occurred alongside low debt (1972-1980), an episode that my model cannot fully explain. My model is designed to account for scenarios of high debt with high inflation or high debt with low inflation. High inflation during "good times" is attributed in my model to a significant inflationary/monetary shock, rather than any fundamental economic decisions by either government type.

The rest of the paper is organized as follows: in Section 3, I present the basic structure of the framework I consider, together with the assumptions I will be imposing. Then, to fix ideas and generate a benchmark that is helpful to fully understand my model, I present a dynamic game in which government reputation has no role. In Section 5, I present my reputation model together with the main results of the paper. Section 6 discusses how I use the model to study the debt and inflation recent history of four emerging market economies (Mexico, Colombia, Guatemala, and Thailand), together with the model's predictions about inflation expectations and government reputation in each of these countries.

2 Related Literature

This paper builds on the foundational monetary models that offer various positive theories for inflation. The literature began with Barro and Gordon (1983a), where high inflation rises since private agents understand the government's incentives to surprise them with inflation in order to increase revenue and reduce unemployment. To prevent this, agents set their inflation expectations higher than the ideal level, leading to a situation where inflation is persistently higher than desired, but no surprise inflation occurs. As a result, monetary policy becomes ineffective in influencing unemployment or output. Canzoneri (1985) stresses the relevance of private information, pointing out that this incentive to "inflate away" may be worse whenever the government possesses private information (for example a forecast on money demand): there are scenarios in which, given its private information, the Monetary Authority may induce some inflation that from the point of view of private agents may be not ideal, which makes agents have even higher inflation expectations. This divergence of information between agents, generates also a credibility problem for the government when instrumenting its policies.

More recent work, for example Bassetto and Miller (2022), also explains inflation as a consequence of differing incentives between the government and private agents. However, they incorporate fiscal considerations into their analysis. Their paper explores how fiscal deficits can sometimes lead to inflation. The key idea is attention: when inflation is high, agents are more motivated to gather information about economic fundamentals, which leads them to adjust their expectations based on this information, translating high deficits into elevated inflation. Conversely, when inflation is low, agents pay less attention, making their expectations less responsive to increases in fiscal deficits.

My paper contributes to this literature by proposing a theory that explains the relationship between debt and inflation, particularly why these variables are more closely correlated in some periods than in others. Similar to Barro and Gordon (1983a), my model shows that inflation increases because agents understand the government's incentive to "inflate away" wages. However, my model also considers the value of debt. Private agents recognize that when debt is higher, the government, which dislikes high debt, has more incentive to use inflation to reduce it. Additionally, when agents are uncertain about the government's type and its true intentions regarding inflation, the equilibrium inflation rate will depend not only on debt but also on their beliefs about the government's commitment to low inflation (i.e., government reputation). In my model, inflation and debt are highly correlated when the government's reputation is low, as agents expect a government with a poor reputation to generate higher inflation as debt rises, compelling the government to act accordingly.

My paper, like the ones previously discussed, is part of the literature that seeks to understand the "Time Inconsistency Problem of Monetary Policy," which refers to the government's incentive to surprise agents with inflation. This area of research began with the seminal work of Kydland and Prescott (1977). Since then, the literature has branched into two strands: one focuses on understanding how both governments and agents behave in the presence of time inconsistency, and the other (such as Barro and Gordon (1983b) and Diaz-Gimenez et al. (2008)) explores solutions to the time inconsistency problem and examines which credible policies can arise in equilibrium. My paper contributes to the former stream by explaining the connection between inflation and debt, which arises because agents are aware of the time inconsistency problem. In my model, government reputation serves as a mechanism that influences this correlation.

This article builds on the reputation literature that originates with Milgrom and Roberts (1982) and Kreps and Wilson (1982). Since then, reputation has been considered a mechanism to address the time inconsistency problem. Recent applications of this idea in policy games can be found in works such as Barro (1986), Backus and Driffill (1985), Phelan (2006), Dovis and Kirpalani (2020), Dovis and Kirpalani (2021), Amador and Phelan (2021), Fourakis (2023), Chatterjee et al. (2023), where the issue of time inconsistency plays a key role. Most of this literature adopts a "reputation by pooling" approach, where the government's discipline comes from the existence of a "behavioral" type that follows an optimal (Ramsey) rule. A "badly behaved" (strategic or opportunistic) government mimics this behavioral type to build its reputation. Most of the equilibrium strategies are then "trigger strategies" built to punish the opportunistic government in case it deviates from the Ramsey rule (Chari and Kehoe (1990)). In addition, as studied by Cripps et al. (2004), in these games reputation effects are necessarily impermanent, meaning agents eventually figure out which type of government they are facing. The main reason of this is that whenever the opportunistic government has a sufficiently high value of reputation it "betrays" agents and starts behaving opportunistically. These combined features of the reputation by pooling strand of the literature are not what I wish to capture in my framework. I consider the "reputation by separation" concept in the spirit of Mailath and Samuelson (2001), where a "good" government tries to prove to agents that it is indeed "good." However, because agents know there's a chance they are dealing with a "bad" type, it can sometimes be difficult for the "good" type to demonstrate its true nature, depending on the current state of government reputation. I believe this approach better reflects the situation in many emerging markets, where governments committed to low inflation must convince agents that they are not like the "irresponsible" regimes of the past that led to high debt and inflation. This dynamic is particularly relevant to Latin America, as highlighted by Kehoe and Nicolini (2021) in their comprehensive study of the region's monetary and fiscal history. Moreover, as I explain in the next paragraph, recent literature on the interaction of fiscal and monetary policy follows a similar idea.

The seminal work explaining how fiscal considerations influence inflation is Sargent and Wallace (1981). This and other papers study the consequences of a"fiscal dominance" regime, where the Monetary Authority is forced to induce high inflation and abandon its own objectives when debt becomes excessively high. My paper wishes to stress the role of reputation and abstract from other considerations such as "fiscal dominance", hence, it follows the more recent literature on the interaction between fiscal and monetary policies, such as Kocherlakota (2012), Lopez-Martin et al. (2018), Bassetto and Miller (2022), which highlight the role of inflation expectations in understanding this relationship. The central idea in these frameworks is that inflation expectations (and thus inflation itself) respond to changes in debt. This response occurs either because agents anticipate a "bailout" from the Monetary Authority to the Fiscal Authority (as in Kocherlakota (2012)), or due to a regime change back to the Central Bank being under the influence of the Fiscal Authority (as in Lopez-Martin et al. (2018), Bassetto and Miller (2022)).

This paper proposes a theory to explain the relationship between debt and inflation, where the main driver is government reputation. Of course, other mechanisms could also explain how debt translates into inflation, though they are beyond the scope of this paper. One such mechanism, particularly relevant for some emerging economies, is default: where a government denominates its debt in foreign currency and may choose to repay only part of it, or none at all. Studies like Bassetto and Galli (2019), Fourakis (2023) examine how the risk of default affects government behavior and monetary policy. The probability of default influences the interest rates on government debt, which in turn impacts local price formation. My model does not consider this possibility, making it best suited to describe an economy which debt is mostly denominated in its domestic currency.[1] Additionally, my paper fits into the literature on the effects of nominal debt, which can be reduced by inflation. In contrast, indexed debt, which promises a fixed real income to lenders, is neutral to monetary policy. Diaz-Gimenez et al. (2008) study both nominal and indexed debt in the context of time inconsistency and find that nominal debt leads to worse outcomes in terms of welfare and higher inflation. Finally, this paper focuses on how inflation reacts to "fundamental" fiscal policy changes, while abstracting from reactions to "non-fundamental" changes, such as the explosive price spirals discussed by Sargent et al. (2009) or hyperinflation models like Cagan (1956).

3 Preliminaries

I consider a game between two players: a continuum of monopolistically competitive wage setters and a (consolidated) government. Time is discrete and has an infinite horizon. In each period wage setters decide their individual wage w_t^i, which in turn determines the aggregate average wage w_t, and, simultaneously, the government chooses the money supply m_t (which in turn determines the price level p_t), current deficit level d_t, and debt b_t. These decisions pin down output:

$$y_t = \bar{y} + \theta \left(\frac{p_t - w_t}{p_{t-1}} \right) + d_t,$$

where $\bar{y}$ is the natural level of output, and $\theta > 0$. This equation aims to capture the idea that output fluctuates around a natural level, and these variations are driven by the labor market or by government intervention. In the labor market, since wages are fixed from the point of view of firms, a higher price attracts more firms to the market, increasing employment and output. In this sense, the parameter θ can be interpreted as the incidence of the labor market on output. On the other hand, a higher deficit, i.e. higher expenditures relative to taxes, also generates more production.

3.1 Wage Setters

Following the seminal literature on monetary policy games, e.g. Fischer (1977) and Canzoneri (1985), I assume that wages must be set in a labor contract prior to the setting of the money supply, and thus the realization of the price level. Each individual wage setter $i \in [0, 1]$ seeks to set a wage that will guarantee a constant level of consumption over time. This can be achieved if the evolution of the real wage that workers receive stay constant. To capture this, the payoff of wage setter i is given by:

$$UW_t^i = -\left(\frac{w_t^i - p_t}{p_{t-1}} \right)^2.$$

These payoffs have the following additional interpretation: if we use the equation that deter-

1 The countries I consider, with the exception of Mexico during 1982-1987, have more than 50% of its debt denominated in their domestic currencies.

mines output and define $\tilde{y}_t = y_t - d_t$, then:

$$UW_t^i = -\frac{1}{p_{t-1}^2}(w_t^i - p_t)^2 = -\frac{1}{p_{t-1}^2\theta^2}\left(y_t - d_t - \bar{y}\right)^2 = -\frac{1}{p_{t-1}^2\theta^2}\left(\tilde{y}_t - \bar{y}\right)^2.$$

Since the natural level of output $\bar{y}$ is associated with the natural level of unemployment, I am modelling wage setters as choosing their wage in order to target the natural level of unemployment in the current period (since they do not decide d_t, the best thing wage setters can do is to make $\tilde{y}_t$ as close as possible to $\bar{y}$).

From the wage setters' point of view, the price level p_t is taken as given at the time of deciding their wage, and, therefore, wage setters will choose their wage based on their expectation of the price level. The expected utility-maximizing strategy for wage setters is thus:

$$w_t^i = p_t^{e,i},$$

where $p_t^{e,i}$ is the prediction (expectation) of the price level of wage setter $i \in [0,1]$. Since every wage setter will have the same information available each period, every wage setter i chooses the same wage, which in turn implies that the average wage is $w_t = w_t^i = w_t^j$ for all $i, j \in [0,1]$. Hence, from now on I focus on the determination of w_t considering the problem that a representative wage setter chooses. Defining $\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}}$, and $\pi_t^e = \frac{p_t^e - p_{t-1}}{p_{t-1}}$ then we can re-write the payoffs of wage setters as well as the output equation as:

$$UW_t = -(\pi_t - \pi_t^e)^2,$$

$$y_t = \bar{y} + \theta(\pi_t - \pi_t^e) + d_t.$$

This results in a familiar prediction error model for the wage setters. From this point onward, I will discuss the results of the model referring to the wage setters as choosing π_t^e instead of w_t.

3.2 Consolidated Government

I consider a government that decides on both the fiscal and monetary policies it implements. Hence, I am consolidating the fiscal and monetary authorities into a single institution. The government chooses (m_t, d_t, b_t) having three objectives in mind: inducing an output level close to an exogenously determined target ($k\bar{y}$), inflation being close to an exogenously given target ($\bar{\pi}$), and to not have an exploding debt level. The flow-payoff of the government is given by:

$$UG_t = -(y_t - k\bar{y})^2 - s(\pi_t - \bar{\pi})^2 - \gamma b_t^2,$$

where $k > 1, s > 0, \gamma > 0$ and $\bar{\pi} > 0$. The first two terms are seminal in monetary games, such as the one discussed by Canzoneri (1985), and they capture the idea that the government's objective differs from the one of the wage setters in two ways: first, the government aims to induce an output level higher than the one desired by the wage setters ($\bar{y}$), motivated by the fact that $\bar{y}$ implies an unemployment level that is too high from the government's perspective. Second, the government cares about inflation not being too far from its target. As the parameter s is larger, the higher the punishment for government payoffs are for allowing large deviations of inflation from its target.

Finally, the last term in the government's flow-payoffs implies that a higher debt will lower the utility of the government. There are several reasons explored in the literature for why governments dislike having high debt. For example, in the sovereign default literature, e.g. D'Erasmo

(2011) and Amador and Phelan (2021), governments are concerned about having a high debt since this leads to a higher probability of default and therefore a decrease in the perception that the government is committed to not default. The political economy literature, e.g. Alesina and Tabellini (1990), assumes that governments dislike high debt since this may lead to a higher probability of a fiscal crisis, which in turn hurts the chances of government re-election. In this model, I abstract from all these potential considerations for the government, making the reduced-form approach that government receives a lower payoff as debt increases.

The government faces the following budget constraint in real terms:

$$b_t = d_t + \frac{(1+i_t)b_{t-1}}{1+\pi_t} - S_t,$$

where i_t is the nominal interest rate and S_t represents the seigniorage generated by the government. I assume that in this world the Fischer equation holds, which means that $1 + i_t = (1+r)(1+\pi_t^e)$, where $r > 0$ is a parameter of the model that represents the "natural" interest rate. Hence, if inflation expectations are higher than inflation, this increases the real interest rate that the government must pay for its current debt. From the point of view of the government inflation expectations are given, so the government could allow for higher inflation in order to reduce the real interest rate of its current debt.

Given that the focus of this paper is to explore situations in which a government generates debt and is not a lender, I restrict debt to be non-negative. Also, for technical reasons that simplify things in the equilibrium existence proof, I assume that debt is bounded above by $\bar{b} > 0$. I restrict the parameters of the model such that, in equilibrium, it is always the case that debt decisions are not binding (i.e. they are never zero nor $\bar{b}$). If debt decisions were binding at some point then the economy would be in a fiscal dominance situation, in the spirit of Sargent and Wallace (1981), in which inflation must be used to reduce debt in order for it to be feasible, and other monetary policy considerations become of second order. Since this scenario is not part of the scope of this paper, I impose the following parameter restrictions that guarantee that debt decisions are always interior.

Assumption 1. In order for the government debt decisions to be such that $b' < \bar{b}$ in the stage game presented in this section, it must be the case that:

$$1 + r - 2\bar{m} < 0.$$

In the rest of the paper, I assume that this parameter restriction is satisfied. Appendix A provides details on how this restriction is attained and how, if assumed, it guarantees that debt decisions are interior. Another assumption that I make throughout the paper has to do with the relationship between the discount factor δ and the natural interest rate r. Since I focus on scenarios in which the government is a borrower, I require that the natural interest rate is such that the government would like to borrow if it were to be facing a real interest rate of r (the actual value of the real interest rate that the government pays is determined in equilibrium).

Assumption 2. In order for $b' > 0$ in the stage game presented in this section, it must be the case that:

$$r < \frac{1-\delta}{\delta}.$$

In other words, the natural interest rate should be smaller that the implied discount rate by δ. If both these assumptions are satisfied, then government decisions are guaranteed to rely on $[0, \bar{b}]$.

Finally, in order to close up the model, I need to discuss how prices are determined. This will be done in a very simplistic way, by introducing a simple money demand equation:

$$\frac{m_t}{p_t} = \bar{m},$$

where $\bar{m} > 0$. This implies that real balances are constant, and therefore, the government can choose the growth level of the money supply g_t in order to pin down the inflation rate π_t. From now on, I will be discussing the results of the model in terms of the government choosing π_t instead of m_t.

Notice that having a constant real balances demand also implies that seigniorage becomes $S_t = \bar{m}\pi_t$.[2] This implies that seigniorage is an increasing function of inflation, meaning that an elevated inflation generates higher revenue for the government associated to money printing.

With all of these considerations, the government's budget constraint can be re-written as:

$$b_t = d_t + \frac{(1+r)(1+\pi_t^e)b_{t-1}}{1+\pi_t} - \bar{m}\pi_t.$$

Hence, debt will tend to increase if the fiscal deficit of the government increases, but as inflation raises, the debt will become lower both via a lower real interest rate and higher seigniorage.

3.3 Static Nash Equilibrium

Now, I turn to analyze the equilibrium that would arise in the case that these agents were to play the previously described game once. The solution concept that I use is Nash equilibrium. In this game, there is a state variable: the debt level inherited by the government, $b = b_{t-1}$. In this sense, the equilibrium is a strategy profile $\left(\pi^{e\star}(b), (\pi^\star(b), d^\star(b), b'^\star(b))\right)$ such that given the strategy of the government, $\pi^{e\star}(b)$ is a best-response for wage setters, and given the strategy of wage setters, $(\pi^\star(b), d^\star(b), b'^\star(b))$ are best responses for the government. The best response for wage setters given the government's strategy $(\pi(b), d(b), b'(b))$ is:

$$\pi^{e,BR}(b) = \pi(b),$$

while the government's best response to $\pi^e(b)$ is characterized by being the (implicit) solution to:

$$\theta\left((1-k)\bar{y} + \theta(\pi - \pi^e(b)) + d\right) + \pi - \bar{\pi} - \gamma\left(\frac{(1+r)(1+\pi^e(b))b}{(1+\pi)^2} + \bar{m}\right)\left(d + \frac{(1+r)(1+\pi^e(b))b}{1+\pi} - \bar{m}\pi\right) = 0,$$

$$(1-k)\bar{y} + \theta(\pi - \pi^e(b)) + d + \gamma\left(d + \frac{(1+r)(1+\pi^e(b))b}{1+\pi} - \bar{m}\pi\right) = 0.$$

$$b' = d + \frac{(1+r)(1+\pi^e(b))b}{(1+\pi} - \bar{m}\pi.$$

2 Let m_t, p_t denote the money and price level, respectively. Seigniorage is given by $S_t = (m_t - m_{t-1})/p_t$. Since $m_t/p_t = m_{t-1}/p_{t-1} = \bar{m}$, then $S_t = \pi_t m_{t-1}/p_{t-1} + (m_t/p_t - m_{t-1}/p_{t-1}) = \bar{m}\pi_t$.

As shown in Appendix A, these set of equations have a unique solution, as well as there is a unique static Nash equilibrium in pure strategies. The following proposition characterizes equilibrium behavior.

Proposition 1. *In the static Nash equilibrium* $(\pi^{e\star}(b), (\pi^\star(b), d^\star(b)))$ *of this game:*

1. $\pi^\star(b)$ *is an increasing function of* b*.*
2. $d^\star(b)$ *is a decreasing function of* b*.*
3. *Wage setters' payoffs in equilibrium are zero, while the government's payoffs are decreasing in* b*.*

The proof of this proposition can be found in Appendix A. In equilibrium, if the government enters the game with a high debt level, since it dislikes having an elevated debt, it will try to decrease it by having both an elevated inflation and a lower deficit. Anticipating this, wage setters increase their inflation expectations as a function of b. Also, in equilibrium there is no surprise inflation, meaning $\pi^{e\star} = \pi^\star$, so inflation cannot be used as a mechanism to boost output nor to reduce the real interest rate paid by the government, since $(1+r)(1+\pi^e)/(1+\pi)$ is equal to $1+r$ in equilibrium. Hence, the only reason why the government wants higher inflation in equilibrium is because this generates larger seigniorage, which helps reduce debt. In terms of welfare, higher current debt is not desirable for the government, since it dislikes having a huge future debt b'. Hence, whenever debt is high the government must use inflation and deficit to reduce debt, but this has a negative impact on output and generates an inflation further away from its target. Then, government welfare is decreasing on b. On the wage setters side, since there is no surprise inflation, their welfare is equal to zero.

This equilibrium is static in the sense that agents are not internalizing how their choices will affect the evolution of debt and, hence, their future payoffs. The following section of the paper presents a game in which the government will not only care about its current payoffs, but also about future ones, which will force it to internalize how its choices affect the evolution of debt and other variables.

4 Benchmark Model: Dynamic Game

Before I fully delve into my reputation framework, as a preliminary step, I present a model in which the government and the wage setters interact repeatedly over time without reputational concerns. This allows me to present definitions that will be helpful in the reputation framework, but also to justify some of the assumptions I will be imposing on my model.

Since there is a continuum of wage setters, each of them understands that their wage decision does not affect the aggregate wage level nor the evolution of wage through time, and therefore will act as myopic players (or short-lived).[3] On the other hand, the government is a long-lived player that takes into account the consequences of its actions on the future, and has a discount factor $\delta \in (0,1)$.

This framework consists of a perfect monitoring dynamic game, in which there is a long-lived player (the government) and a myopic player (the wage setters). The flow-payoffs and available actions for both players are as described in the previous section, however, it is now moment to highlight that the payoffs of the government are affected by the previous debt level

3 The fact that the government also cares only about the aggregate wage and not about the individual decision of each wage setter is also relevant for this result. For more on this, consult Fudenberg et al. (1998).

b_{t-1}, and that the government's decision on b_t will affect the government's future payoffs. Hence, debt is a state variable in this game.

Timing in this framework is as follows: in each period t, upon observing the history of previous play, wage setters choose their inflation expectations π_t^e and the government simultaneously chooses (π_t, d_t, b_t). At period t, a history of play, h^t, is given by:

$$h^t = (b_0, \pi_1, \pi_1^e, d_1, b_1, ..., \pi_{t-1}, \pi_{t-1}^e, d_{t-1}, b_{t-1}).$$

In general, players' strategies may be a complicated function of the preceding history of play. Following the literature on dynamic games, e.g. Mailath and Samuelson (2006), Phelan (2006), I focus on homogeneous and stationary Markov strategies (that in the rest of the paper I will just call Markov strategies), which are a function of the current state variable, in this case the value of b_{t-1}.

I denote $\sigma_w : \mathcal{D} \to \mathbb{R}$ to be the strategy for wage setters, where $\sigma_w(b)$ is the inflation expectation chosen by wage setters when the previous debt level is b. The domain of this strategy, $\mathcal{D} = [0, \bar{b}]$ considers that debt cannot be negative and is bounded above by a parameter $\bar{b} > 0$. I consider the same parameter restrictions as discussed in the previous section such that the upper bound on debt is not binding. I denote σ_G to be the strategy for the government, where

$$\sigma_G(b) = \left(\pi(b), d(b), b'(b)\right),$$

is the vector of choices made by the government when the previous debt level was $b \in \mathcal{D}$.

The wage setter's best reply to σ_G is characterized by the following problem:

$$\sigma_w(b) = \text{argmax}_{\pi^e} \; -\left(\pi^e - \pi(b)\right)^2,$$

implying that $\sigma_w(b) = \pi(b)$ for all $b \in \mathcal{D}$. Notice that since debt is a state variable, the best reply of wage setters is a function $\sigma_w : \mathcal{D} \to \mathbb{R}$.

On the other hand, the government's best reply to σ_w is characterized by the following dynamic programming problem:

$$\begin{aligned} V(b) = \max_{\pi,d,b'} \;\; & (1-\delta)\left[-(y-k\bar{y})^2 - s(\pi-\bar{\pi})^2 - \gamma(b')^2\right] + \delta V(b'), \\ & y = \bar{y} + \theta\left(\pi - \sigma_w(b)\right) + d, \\ & b' = d + \frac{(1+r)(1+\sigma_w(b))b}{1+\pi} - \bar{m}\pi, \\ & 0 \leq b' \leq \bar{b}. \end{aligned} \tag{1}$$

Notice that σ_w affects the government's choices in three ways: first, it influences the current period's output; second, it affects the nominal interest rate; and third, the functional form of σ_w affects the continuation payoff of the government. In the current period the government must evaluate how any potential inflation and deficit decisions translate into a new debt, which in turn determines inflation expectations in the following period. Then, in equilibrium, the government needs not only to understand how expectations are formed given today's debt level b, but they need to infer how wage setters will react to any b'. In this sense, the best reply to σ_w of the government is a triplet of functions $\pi : \mathcal{D} \to \mathbb{R}$, $d : \mathcal{D} \to \mathbb{R}$, $b' : \mathcal{D} \to \mathbb{R}$.

4.1 Equilibrium: Definition and Existence

In this section and in the rest of the paper, I focus on characterizing Markov perfect equilibria. I also require the equilibrium function σ_w to be continuous, differentiable,, and to have a uniformly bounded first derivative. This assumption, which is purely technical, allows me to use the machinery of optimal control theory to characterize equilibrium behavior in my model.

Definition 1. A Markov perfect equilibrium of this dynamic game is a strategy profile (σ_w, σ_G) such that:

1. (σ_w, σ_G) are Markov strategies.
2. Taking σ_w as given, σ_G solves Equation (1).
3. Taking σ_G as given, wage setters find σ_w to maximize their payoffs.

Notice that, in this definition, the perfection refinement of equilibrium relies on the fact that I am requiring σ_G to solve the government's recursive problem for any possible value of current debt. If there is a deviation from one of the players at any point in time, this changes the observed debt path. Even after this deviation, I am requiring the government to respond to this deviation optimally.

The following result characterizes the existence of a Markov perfect equilibrium in this dynamic game.

Theorem 1. *A perfect Markov equilibrium of this repeated game exists. Furthermore, in any equilibrium it must be the case that:*

$$\sigma_w(b) = \pi(b) \quad \text{for all } b \in \mathcal{D}.$$

The second part of the theorem should not be surprising: it states that in any equilibrium, the inflation expectations of wage setters given b must be equal to the inflation chosen by the government given b. In other words, as the literature would call this, any equilibrium in this framework must be a *rational expectations equilibrium*. This characterization is a consequence of the assumed payoffs for wage setters, since their only goal is to minimize the distance between inflation and expected inflation. In other words, σ_w (as a function) must be a fixed point of the best-reply mapping $BR_G : \Sigma \to \Sigma$ that maps $\sigma_w(\cdot)$ into $\pi(\cdot)$.[4] The proof of this theorem, which is in essence an application of the Schauder Fixed-Point Theorem, provides details on how to guarantee that indeed this mapping has a fixed point. This proof can be found in Appendix B.

In terms of uniqueness, for the moment all I can say (considering evidence from the numerical exercises I have conducted) is that it appears that there is a unique Markov perfect equilibrium, as it is the case in most of the papers that use this type of equilibrium restrictions.[5]

Conjecture 1. *There is a unique Markov perfect equilibrium.*

4 Σ is the set of differentiable functions with domain $\mathcal{D}$ whose derivatives are uniformly bounded. See Appendix B for more details.

5 In order to solve the model numerically I am using an iterative method, described in Appendix D, which requires an initial guess for σ_w. I have tried different initial guesses, and in all cases the algorithm converges to the same equilibrium, which is why I conjecture uniqueness.

4.2 Equilibrium Characterization

The following results aim to characterize some qualitative properties of the equilibrium in this model, in order to have some intuition on how the government responds to changes in b and other model parameters.

Proposition 2. *The following properties hold on-path of every Markov perfect equilibrium* $(\sigma_w^\star, \sigma_G^\star)$ *of this dynamic game:*

1. *No surprise inflation, i.e.,* $\sigma_w(b) = \pi(b)$ *for all* $b \in [0, \bar{b}]$.
2. *The real debt evolution is given by:*

$$b'(b) = d(b) + (1+r)b - \bar{m}\pi(b).$$

3. *Output is given by:*

$$y(b) = \bar{y} + d(b).$$

In this equilibrium, the optimal inflation rate for the government is a fixed point of the best reply function $BR_G : \Sigma \to \Sigma$ which takes as given $\sigma_w(\cdot)$ and returns $\pi(\cdot)$. Hence, in equilibrium inflation expectations of wage setters are equal to the inflation choice of the government. As a consequence, the real interest rate, which is given by $(1+r)(1+\sigma_w(b))/(1+\pi(b))$, is equal to $1+r$, implying that there is no way for the government to affect the real interest rate in equilibrium. This also has an impact on the evolution of output, since inflation does not have a short run impact on GDP, only fiscal deficits can actually affect output. Inflation is then only beneficial for the government through seigniorage, since higher inflation does translate into higher seigniorage in equilibrium.

The following proposition characterizes equilibrium behavior and payoffs.

Proposition 3. *In every Markov perfect equilibrium* $(\sigma_w^\star, \sigma_G^\star)$ *of this dynamic game:*

1. V *is a continuous, decreasing, strictly concave, and differentiable function of* $b \in [0, \bar{b}]$
2. *Wage setters have a payoff of zero for all* $b \in [0, \bar{d}]$.
3. $\sigma_w^\star$ *is an increasing function of* $b \in [0, \bar{d}]$.
4. π *is an increasing and differentiable function of* $b \in [0, \bar{d}]$.
5. d *is a decreasing and differentiable function of* $b \in [0, \bar{d}]$.
6. *Let* $s > s'$. *Then,* $\pi(\cdot|s) \leq \pi(\cdot|s')$ *for all* $b \in [0, \bar{d}]$.
7. *Let* $k > k'$. *Then,* $\pi(\cdot|k) \geq \pi(\cdot|k')$ *for all* $b \in [0, \bar{d}]$.

The proof of this proposition can be found in Appendix B. Intuitively, high debt implies that if the government does not lower it, it will suffer a loss in utility. In order to reduce debt, the government must either decrease deficit (which generates lower output), or increase inflation (which may deviate inflation from its target). Both scenarios are not desirable by the government, and hence its value function is decreasing in b. Inflation is an increasing function of debt since, as b gets higher, wage setters understand the temptation of the government to erode debt with higher inflation, and hence they increase their inflation expectations (wage demand), which in turn boost inflation up.

Regarding deficit decisions, for positive b's, the government always wants to decrease deficit whenever debt increases, given the disutility of high debt. In terms of future debt, in equilibrium, the on-path debt will evolve according to:

$$b' = d + (1+r)b - \bar{m}\pi,$$

since $\sigma_w(b) = \pi(b)$. In general, as b increases, there is an ambiguous effect on b' since, on the one hand, inflation is increasing, which reduces the debt via higher seigniorage; but, on the other hand, b itself is increasing, so the effect on the service of the debt $(1+r)b$ is going up. This is the reason of why I cannot characterize in general the evolution of future debt as a function of b.[6]

Finally, the last two results are intuitive: a higher s, which can be interpreted as a higher commitment of the government to its inflation target, implies that the government wants to have lower inflation; while a higher k, which increases incentives to generate surprise inflation, implies that the government chooses an elevated inflation for it to boost output.

In essence, in this Markov equilibrium, the government wants to keep debt as low as possible, since this crucially determines both their flow-payoffs as well as their continuation value (which decreases with higher debt). As a consequence, inflation is higher when b increases, and deficit is lower.

4.3 Equilibrium Dynamics

In this section I present some results from a numerical solution of the model, in order to further highlight the intuition behind it. The parameters considered, which are displayed in Table I, are calibrated in order to match some features of the Mexican data between 2000-2022.[7] The details on this numerical implementation can be found in Appendix D.

Table I: PARAMETER VALUES.

Parameter	**Interpretation**	**Value**
$\bar{y}$	Natural Level of Output	1
θ	Sensitivity of Output to Inflation	0.5
k	Time Inconsistency Parameter	2
s	Deviations From Inflation Target Weight	10
γ	Debt Weight	2
$\bar{\pi}$	Inflation Target	3%
r	Interest Rate	5%
δ	Discount Factor	0.9

This section aims to highlight the crucial role of two parameters (which will be later exploited in the reputation model section): the discount factor δ, and the weight on the government's utility of inflation deviations from its target s. Intuitively, the discount factor is relevant to determine the evolution of debt since higher discount factors mean that the continuation value for the government has a higher weight on its current utility. Hence, if we were to compare two governments that only differ in the discount factor, the one that has a higher discount factor

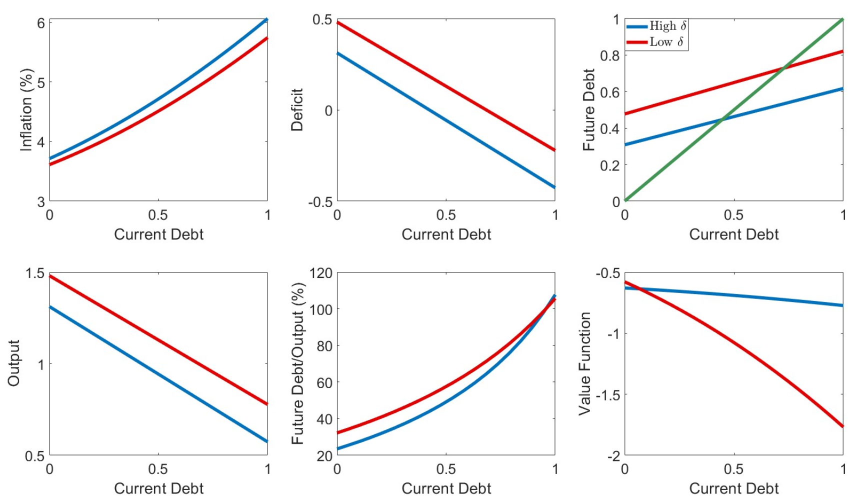
6 Nevertheless, most of the simulations I present throughout the paper have $b'(b)$ being an increasing function of b. This is due to the sum of primary fiscal deficit plus real debt repayment being positive.

7 More details on the calibration procedure can be found in Section 6.3.

should produce a series of debt over time that is lower. On the other hand, the parameter s has a direct impact on the inflation level, in the sense that a government with higher s will have an inflation time series that is closer to the inflation target.

Figure II presents the equilibrium strategies for two different governments: one with a low discount factor ($\delta = 0.1$), and the other with a high discount factor ($\delta = 0.9$). As the government puts a higher weight into the continuation value on its current payoffs (higher δ), the government will be more concerned about having a higher debt, since in general higher debt leads to lower continuation payoffs. Hence, as shown in the third panel of this figure, a government with higher discount factor will tend to have lower future debt (b') for every value of current debt (b). Both government's decisions will converge to a steady state, with the more patient government having a lower steady state debt. Having a more controlled debt is what allows the government with higher δ to achieve higher payoffs, especially when the current debt is high.

Figure II: EQUILIBRIUM WITH DIFFERENT DISCOUNT FACTORS.

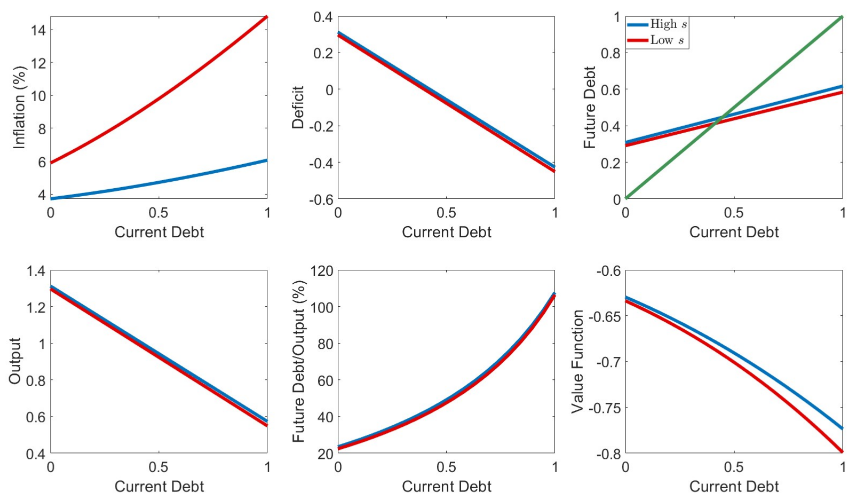


NOTES: This graph plots the equilibrium functions considering the values presented in Table I, for $\delta = 0.1$ (red line) and $\delta = 0.9$ (blue line). The green line on the third panel represents the 45 degree line.

This figure also highlights that if I only consider variations in δ, this has an impact on the evolution of inflation. Since governments with higher discount factor tend to care more about debt, they will allow higher inflation. This is because higher inflation is a "cheaper" way to reduce inflation relative to reducing fiscal deficit: since surprise inflation in equilibrium is zero, higher inflation does not impact output but does reduce future debt through seigniorage. On the other hand, reducing fiscal deficit has a negative impact on output. Then, it is less costly for the government to reduce debt using inflation than through reducing fiscal deficits.

Figure III presents the equilibrium functions whenever I consider two different governments that only differ in the weight they give to inflation deviations from its target, i.e., on the value of the s parameter. As we can see in the first panel of this figure, a government with a higher s produces an inflation sequence that is lower, and closer to its target. In contrast to what happened in the previous exercise, this government (the one with higher s) has a lower seigniorage revenue, and hence the evolution of debt will be slightly higher. In fact, a government with a higher value of s has a slightly higher value of steady state debt (this can be seen in the third panel of the figure), and has a slightly higher quotient of debt relative to output.

Figure III: EQUILIBRIUM WITH DIFFERENT VALUES OF s.

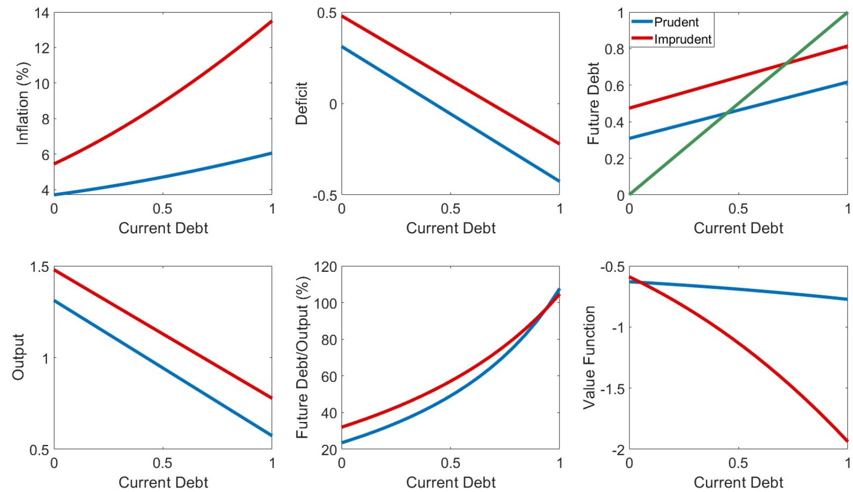


NOTES: This graph plots the equilibrium functions considering the values presented in Table I, for $s = 1$ (red line) and $s = 10$ (blue line). The green line on the third panel represents the 45 degree line.

Finally, Figure IV presents the equilibrium behavior for two different governments: one that has both a high discount factor ($\delta = 0.9$) and a high value of s ($s = 10$), which I refer to as the prudent government; and a government that has a low discount factor ($\delta = 0.1$) and a value of $s = 1$, which I refer to as the imprudent government. As a result of varying these two key parameters, the prudent government will generate an equilibrium sequence of both debt and inflation that is lower. Hence, in order for this model to be able to predict government behavior such that both debt and inflation in steady state are low, it is necessary to ask that the government values both the continuation value (and hence the evolution of debt) as well as the inflation deviations from its target. I use this insight in the following section to define the two types of governments I will be analyzing.

In summary, this dynamic game generates behavior that allows us to understand how a government could be generating low debt and low inflation scenarios (by being prudent) or high inflation with high debt. If we were to believe this model to be a description of the data, one should expect then a high correlation between debt and inflation. However, the data of several emerging countries suggest that during prolonged time spans, inflation and debt seem not to be that correlated. This is something that the dynamic model presented cannot fully account for. This is the reason of why I introduce an additional consideration to the model, which will be incomplete information about the government's type, and by doing this, the model will be able to account for low inflation with high debt episodes.

Figure IV: EQUILIBRIUM WITH PRUDENT AND IMPRUDENT GOVERNMENTS.



NOTES: This graph plots the equilibrium functions considering the values presented in Table I, for an imprudent government, which has a discount factor $\delta = 0.1$, and $s = 1$ (red line), and for a prudent government, which has a discount factor $\delta = 0.9$, and $s = 10$ (blue line). The green line on the third panel represents the 45 degree line.

5 Reputation Framework

As motivated in Section 2, recent literature on the interaction between fiscal and monetary policies highlights the inflation expectations channel as important in order to understand how inflation and debt are correlated. Whenever debt is high, agents know that the government will have to generate higher inflation in order to dilute the service of the debt, so they expect to experience an elevated inflation rate. As a result, there is a high correlation between inflation, deficit, and debt. Whenever the government is perceived to be committed to its inflation target, agents' inflation expectations are less responsive to changes in debt, and therefore the correlation between inflation and debt decreases. The reputation framework I present in this section aims to capture this intuition.

This framework has the same players and flow-payoff structure that the dynamic game presented before. However, in this game, wage setters have uncertainty regarding which type of government they are facing. The government can be one of two types:

1. Type ξ^P (Prudent): a government that has a discount factor $\delta_P \in (0, 1)$, and a parameter $s = s_P > 0$.

2. Type ξ^I (Imprudent): a government that has a discount factor $\delta_I = 0 < \delta_P$, and a parameter $s = s_I < s_P$.

Let $\rho_0 \in [0, 1]$ be the prior probability that government is of type ξ^P. I interpret ρ_0 and its updates through time as the government's reputation. As discussed in Section 4.3, governments that have lower δ and lower s tend to generate both a higher debt and inflation. Then, a government of type ξ^I tends to generate higher debt and inflation than type ξ^P. I decided to model a government having these particular types since I want to capture the idea that if agents observe inflation and debt being elevated for a couple of periods, they will believe their are facing a government which is not committed to generate low inflation.

In typical reputation frameworks (e.g., Backus and Driffill (1985)), it is common to assume that one type of government behavioral (it follows a predetermined strategy). This assumption lends tractability to the model and provides a clear interpretation of the probability that agents assign to facing a behavioral type (for instance, if the behavioral type follows a rule, this probability reflects the likelihood that the government is adhering to that rule). However, since my model includes a state variable (debt) that evolves over time, I require greater flexibility in the government's behavior to account for this state. In my game, both government types are strategic. Nonetheless, by making ξ^I myopic, it does not internalize how its decisions affect either the evolution of debt or its future reputation. This simplification ensures that its behavior remains straightforward enough to characterize, thereby preserving a decent level of tractability for both the model and the proofs.

There is an additional element that I must introduce for to the model to generate non-trivial reputation dynamics: imperfect monitoring.[8] To introduce imperfect monitoring without loosing tractability, I assume that the materialized (observed) level of inflation, deficit, and debt are the result of the government's choices plus some noise. This is a reduced-form approach to incorporate other factor that could be relevant to determine these variables and that are not in the direct control of the government (e.g. exchange rate fluctuations, international shocks, etc...). Conditional on the government being of type ξ, in every period wage setters observe $(\hat{\pi}, \hat{d}, \hat{b})$, where:

$$\hat{\pi} = \pi^{\xi} + \epsilon_{\pi}, \;\; \hat{d} = d^{\xi} + \epsilon_d, \;\; \hat{b} = \hat{d} + \frac{(1+r)(1+\pi^e)b}{1+\hat{\pi}} - \bar{m}\hat{\pi},$$

where ϵ_x are i.i.d. random variables with mean zero and variance σ_x^2, $x \in \{\pi, d\}$. Hence, a history of play h^t observed by wage setters at period t is given by:

$$h^t = \left(b_0, \hat{\pi}_1, \pi_1^e, \hat{d}_1, \hat{b}_1, ..., \hat{\pi}_{t-1}, \pi_{t-1}^e, \hat{d}_{t-1}, \hat{b}_{t-1}\right).$$

Timing in this framework is as follows: in each period t there are two sub-periods. In the first one, wage setters observe the history of play h^t, updating their beliefs about the government's type, and choose their inflation expectations π_t^e in order to maximize their expected utility. At the same time, the government observes h^t, and chooses (π_t, d_t, b_t). Finally, in the second sub period, shocks materialize and $(\hat{\pi}, \hat{d}, \hat{b})$ are observed by both wage setters and the government.

Once again, I focus on Markov strategies, which are now a function of two variables: the current debt level b, and the current government's reputation ρ. I denote $\sigma_w : [0, \bar{b}] \times [0,1] \to \mathbb{R}$ to be the strategy for wage setters, where $\sigma_w(b, \rho)$ is the inflation expectation chosen by wage setters when the previous debt level was b and the government's reputation is ρ. Similarly, I denote σ_G^{ξ} to be the strategy for the government of type ξ.

Upon observing $(\hat{\pi}, \hat{d}, \hat{b})$ and knowing the current state they are in (b, ρ), wage setters update their beliefs about the government being of type ξ^P following Bayes rule:

$$\rho'(b, \rho) =$$

8 In equilibrium, one of two things can happen: either both government's type pool (i.e. choose the same actions) or separate (i.e. choose different actions). In a perfect monitoring world, if both types pool then private agents will never be able to figure out which type of agents they are facing, and their belief update is constant and equal to the prior; or if both types separate then private agents will immediately know which type of government they are facing.

$$\frac{\rho g_\pi\left(\hat{\pi}-\pi^{\xi^P}(b,\rho)\right)g_d\left(\hat{d}-d^{\xi^P}(b,\rho)\right)}{\rho g_\pi\left(\hat{\pi}-\pi^{\xi^P}(b,\rho)\right)g_d\left(\hat{d}-d^{\xi^P}(b,\rho)\right)+(1-\rho)g_\pi\left(\hat{\pi}-\pi^{\xi^I}(b)\right)g_d\left(\hat{d}-d^{\xi^I}(b,\rho)\right)},$$

where g_π, g_d are the probability density functions of inflation and deficit noise, respectively. Intuitively, this updating rule compares the likelihood of receiving a shock of size $\epsilon_\pi = \hat{\pi} - \pi^{\xi^P}, \epsilon_d = \hat{d} - d^{\xi^P}$ versus facing a shock of size $\epsilon_\pi = \hat{\pi} - \pi^{\xi^I}, \epsilon_d = \hat{d} - d^{\xi^I}$, since wage setters understand the decisions taken by both government types when facing a state (b, ρ).

5.1 Wage Setters' Problem

Considering the previous description of the game, wage setters now choose their inflation expectations in order to maximize their expected utility. Wage setters are uncertain about both the type of government they are facing and the value of shocks $\epsilon^\xi_\pi, \epsilon^\xi_d$. Hence, they decide $\sigma_w(b,\rho)$ in order to maximize their expected utility. In a Markov equilibrium, considering that currently debt is b and government reputation is ρ, and given a conjecture on the government's behavior $(\pi^{\xi^P}(b,\rho), \pi^{\xi^I}(b,\rho))$:

$$\sigma_w(b,\rho) = \text{argmax}_{\pi^e}\ \mathbb{E}_{\epsilon_\pi^{\xi^P},\epsilon_\pi^{\xi^I}}\left[-\rho\left(\pi^e - \pi^{\xi^P}(b,\rho) - \epsilon_\pi^{\xi^P}\right)^2 - (1-\rho)\left(\pi^e - \pi^{\xi^I}(b,\rho) - \epsilon_\pi^{\xi^I}\right)^2\right],$$

which, leads to the following best response for wage setters:

$$\sigma_w(b) = \rho\pi^{\xi^P}(b,\rho) + (1-\rho)\pi^{\xi^I}(b,\rho).$$

Then, wage setters' inflation expectations are a weighted average of the inflation rate that they conjecture a government of each type will choose given the current states, where the weight given to each decision is precisely the current value of government reputation.

This behavior from wage setters, along with the imperfect monitoring assumption, has an important consequence for on-path output and debt, since now it is no longer true that surprise inflation is equal to zero. In this framework, surprise inflation will be given by $\hat{\pi} - \sigma_w(b,\rho)$ which in general is not zero. As a consequence, the gap between realized and expected inflation will have an impact on output and on the real interest rate of debt. In the benchmark model without reputational concerns this effect was not present since surprise inflation was always equal to zero.

5.2 Government's Problem

Taking as given the current value of debt and government reputation, as well as a conjecture on the behavior of a government of type ξ^I, the prudent government's best reply is characterized by the following problem:

$$V^{\xi^P}(b,\rho) = \max_{\pi,d,b'}\ \mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[(1-\delta_P)\left[-(\hat{y}-k\bar{y})^2 - s_P(\hat{\pi}-\bar{\pi})^2 - (\hat{b}')^2\right] + \delta_P V^{\xi^P}\left(b',\rho'\right)\right],$$

$$\hat{y} = \bar{y} + \theta\left(\hat{\pi} - \sigma_w(b,\rho)\right) + \hat{d},$$

$$b' = \hat{d} + \frac{(1+r)(1+\sigma_w(b,\rho))b}{1+\pi} - \bar{m}\pi,$$

$$
\begin{aligned}
\hat{\pi} &= \pi + \epsilon_\pi, \\
\hat{d} &= d + \epsilon_d, \\
\rho' &= \frac{\rho g_\pi\left(\hat{\pi}-\pi\right) g_d\left(\hat{d}-d\right)}{\rho g_\pi\left(\hat{\pi}-\pi\right) g_d\left(\hat{d}-d\right) + (1-\rho) g_\pi\left(\hat{\pi}-\pi^{\xi^I}\right) g_d\left(\hat{d}-d^{\xi^I}\right)}, \\
0 &\le b' \le \bar{b}.
\end{aligned} \tag{2}
$$

In this problem, since the prudent government cares about the future, it has to consider how its current choices of inflation, deficit, and debt will have an impact on both the evolution of debt (as in the benchmark model) but also on the perception of wage setters regarding their beliefs of which type of government they face. As I highlight in the sections below, this creates a trade-off for the government between increasing its reputation and impacting the evolution of debt. Depending on the value of (b, ρ), sometimes it will be beneficial for the government to increase debt (which is costly) and increase its reputation (which is beneficial), or the other way around.

Now, even though the government has to consider additional variables that evolve through time and that can have an impact on their payoffs, when we take as given the behavior of the other agents in this game, as long as the flow payoff function is well behaved (in the sense described in Appendix C), this dynamic problem has still a unique solution which can be characterized using optimal control techniques.

On the other hand, the imprudent government's best reply, taking as given the behavior of ξ^P, is characterized by the following dynamic problem:

$$
\begin{aligned}
V^{\xi^I}(b,\rho) &= \max_{\pi,d,b'} \quad \mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[-(\hat{y}-k\bar{y})^2 - s_I(\hat{\pi}-\bar{\pi})^2 - (\hat{b}')^2\right], \\
\hat{y} &= \bar{y} + \theta\left(\hat{\pi} - \sigma_w(b,\rho)\right) + \hat{d}, \\
b' &= \hat{d} + \frac{(1+r)(1+\sigma_w(b,\rho))b}{1+\pi} - \bar{m}\pi, \\
\hat{\pi} &= \pi + \epsilon_\pi, \\
\hat{d} &= d + \epsilon_d, \\
0 &\le b' \le \bar{b}.
\end{aligned} \tag{3}
$$

Since ξ^I is myopic, it does not internalize how its current decisions of inflation and deficit affect its future reputation. Nevertheless, the impatient government does care about the value of current government reputation ρ, at least indirectly, since this value determines the behavior of wage setters.

5.3 Equilibrium: Definition and Existence

I now define and equilibrium as well as present some arguments that make me belief that a unique equilibrium of this game exists.

Definition 2. A Markov perfect equilibrium of this game is a strategy profile $(\sigma_w, \sigma_G^{\xi^P}, \sigma_G^{\xi^I})$ such that:

1. $(\sigma_w, \sigma_G^{\xi^P}, \sigma_G^{\xi^I})$ are Markov strategies.
2. Taking $(\sigma_w, \sigma_G^{\xi^I})$ as given, $\sigma_G^{\xi^P}$ solves Equation (2).
3. Taking $(\sigma_w, \sigma_G^{\xi^P})$ as given, $\sigma_G^{\xi^I}$ solves Equation (3).
4. Taking $(\sigma_G^{\xi^P}, \sigma_G^{\xi^I})$ as given, wage setters find σ_w to maximize their payoffs.
5. The updating rule ρ' is consistent with Bayes rule for all (b, ρ).

As highlighted before, once I take as given the behavior of other agents in the model, type ξ's problem is similar to the long-lived government of the dynamic game presented in Section 4, so it is still the case that the government's problem has a unique solution. As in other reputation frameworks, showing equilibrium existence requires a "fixed point" argument, and the details of this can be found in Appendix C.

Theorem 2. *A perfect Markov equilibrium of this game exists.*

The following section aims to characterize some aspects of equilibrium behavior.

5.4 Equilibrium Characterization

I first analyze how wage setters' equilibrium behavior changes with debt and government reputation. In equilibrium it is the case that:

$$\sigma_w(b, \rho) = \rho \pi^{\xi^P}(b, \rho) + (1 - \rho)\pi^{\xi^I}(b, \rho) \quad \text{for all } b \in \mathcal{D}, \ \ \rho \in [0, 1],$$

so, inflation expectations are a weighted average of the behavior of the prudent and imprudent government. Given equilibrium behavior in the dynamic game analyzed in the previous section, optimal inflation is increasing in debt, as higher debt can be diluted through seigniorage with higher inflation. Hence, both government types should respond with higher inflation as b increases, which in turn translates into $\sigma_w(\cdot, \rho)$ being an increasing function. On the other hand, since $s_I < s_P$, the imprudent government generates higher inflation than the prudent one, which implies that for a fixed b, as agents belief they are facing a prudent government with higher probability, they should decrease their inflation expectations. Following the same line of thought, if we increase both debt and reputation, since the weight that σ_w gives to π^{ξ^P} is higher, the incidence of changes in b in inflation expectations should be decreasing with ρ. The following theorem formalizes this intuition.

Proposition 4. *The following conditions on wage setters equilibrium behavior holds in every Markov perfect equilibrium of this game:*

1. *For every $\rho \in [0, 1]$, $\sigma_w(\cdot, \rho)$ is strictly increasing in $b \in [0, \bar{b}]$.*
2. *For every $b \in [0, \bar{b}]$, $\sigma_w(b, \cdot)$ is strictly decreasing in $\rho \in [0, 1]$.*
3. *For every $\rho \in [0, 1]$, $b \in [0, \bar{b}]$,*
$$\frac{\partial^2 \sigma_w}{\partial b \partial \rho}(b, \rho) < 0.$$

The last part of this proposition states that as government reputation is higher, we should expect to observe a lower incidence of higher debt on inflation expectations (and, therefore, on inflation). Hence, according to this model, the reason why sometimes we observe episodes of high debt uncorrelated with inflation is due to government reputation being elevated. In Figure I we observe episodes in which debt is highly correlated with inflation, and other times in which there seem to be uncorrelated. My model rationalizes this change in correlation through a change in government reputation: high debt with high inflation episodes occur when government reputation is low, which translates into high inflation expectations and high inflation; while high debt with low inflation can happen when government has high reputation, times in which higher debt has a limited impact on inflation expectations and, therefore, on inflation.

Turning now to government behavior, the optimal inflation of each type turns out to be increasing in debt (as in the dynamic game analyzed in the previous section), while the deficit choice should be decreasing. In terms of reputation, as ρ increases, inflation expectations decrease, and hence there is less room for both government types to generate high inflation rates.

Proposition 5. *The following conditions for both government types' equilibrium behavior holds in every Markov perfect equilibrium of this game:*

1. *For every $\rho \in [0,1]$, $\pi^{\xi}(\cdot,\rho)$ is strictly increasing in $b \in [0,\bar{b}]$ while $d^{\xi}(\cdot,\rho)$ is strictly decreasing.*
2. *For every $b \in [0,\bar{b}]$, both $\pi^{\xi}(b,\cdot)$ and $d^{\xi}(b,\cdot)$ are strictly decreasing in $\rho \in [0,1]$.*

In terms of welfare, in this game the fact that there is another type that wage setters could be facing is detrimental for each government's utility. For the prudent government, the fact that wage setters could be facing an imprudent government makes inflation expectations higher (since they are considering an inflation chosen by the imprudent government that in general is higher), which forces the prudent government to choose higher inflation that what it would choose in the absence of incomplete information (more on this in the following section, Section 5.5). As wage setters are more certain that they are facing the prudent government, inflation expectations are closer to π^{ξ^P}, which is beneficial for the prudent government. A similar story is true for the imprudent government: in general the imprudent government likes to choose high inflation, but as ρ is larger expectations decrease, which limits the capacity for the imprudent government to choose high inflation. Then, the value $V^{\xi}(b,\cdot)$ is an increasing function in reputation for the prudent government, and decreasing for the imprudent government. In terms of debt, both governments dislike higher debt and hence both value functions are decreasing in b. Furthermore, since the flow-payoff function is strictly concave in the relevant variables, $V^{\xi}(\cdot,\cdot)$ is a strictly concave function for both government types. The following proposition presents the results for the behavior of each government's value function, and its proof can be found in Appendix C.

Proposition 6. *The following conditions for both government types' equilibrium behavior holds in every Markov perfect equilibrium of this game:*

1. *$V^{\xi}(\cdot,\cdot)$ is a strictly concave function for all (b,ρ) and $\xi \in \{\xi^P,\xi^I\}$.*
2. *For every $\rho \in [0,1]$, $V^{\xi}(\cdot,\rho)$ is strictly decreasing in b.*
3. *For every $b \in [0,\bar{b}]$, $V^{\xi^P}(b,\cdot)$ is strictly increasing in ρ while $V^{\xi^I}(b,\cdot)$ is strictly decreasing.*

I now bring attention to whether or not a pooling equilibrium of this game can exist, i.e., if in equilibrium it is possible for both types to choose exactly the same values for inflation and deficit, for some values of (b, ρ). Intuitively, this should not be possible given that there is imperfect monitoring: if for some (b, ρ) both governments made the same choices $(\tilde{\pi}, \tilde{d})$, then necessarily one of the two governments has a profitable deviation. Since both governments are pooling, inflation expectations are going to be equal to $\tilde{\pi}$, and there will be no reputational update ($\rho' = \rho$). But then, one of the two governments has an incentive to deviate from $(\tilde{\pi}, \tilde{d})$ since this would not be detected by wage setters (they would interpret the deviation as a negative shock on either ϵ_π or ϵ_d) but brings huge benefits in terms of decreasing future debt. This intuition is captured in the following proposition, and its proof is presented in Appendix C.

Proposition 7. *In every Markov perfect equilibrium of the reputation game, for any (b, ρ), it must be the case that* $\left(\pi^{\xi^P}(b, \rho), d^{\xi^P}(b, \rho)\right) \neq \left(\pi^{\xi^I}(b, \rho), d^{\xi^I}(b, \rho)\right)$.

Then, in equilibrium we should always expect that each government makes a different inflation and deficit choice. However, as discussed in the following section, the difference in the choices of the prudent and imprudent governments will tend to decrease as government reputation is lower. This is due to the fact that whenever ρ is low, the amount of separation required to actually convince wage setters that they are facing a prudent government is so high, that this reputational gain is not worth the cost associated with debt increase.

These last results suggest that the framework I consider is in the class of "Reputation By Separation" literature, as in Mailath and Samuelson (2001), in which agents actually gain utility by showing they are the "good" type and separate themselves from the "bad" type. In this class of frameworks, the imperfect monitoring structure and the fact that there is noise may lead agents to believe that they are facing the "bad" type. In order to prevent this, the "good" type must choose an equilibrium action that, even with noise, allows agents to update their priors in favor of the "good" type. As Section 5.5 discusses, this is exactly what happens in my model with inflation and deficit decisions: the prudent government chooses (π, d) to separate itself from the imprudent government and hence build reputation, even though it comes with the cost of accumulating more debt.

5.5 Equilibrium Dynamics

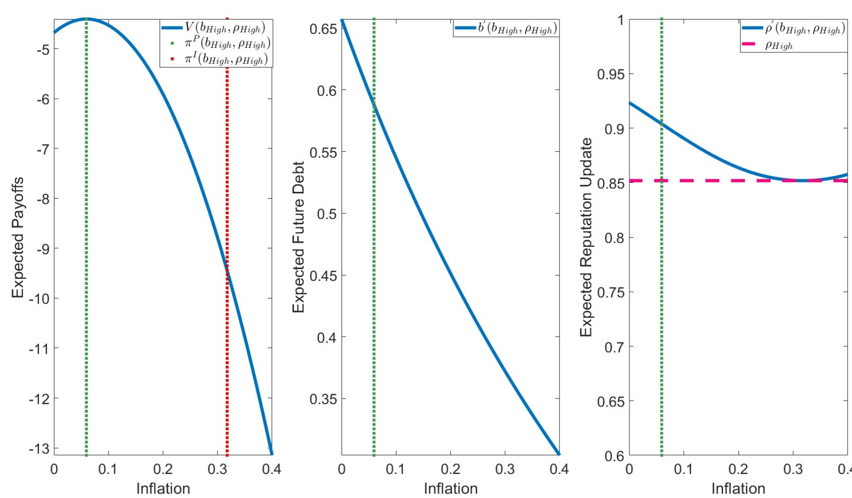
This section presents a numerical solution for the reputation model, with parameters calibrated to match some features of the Mexican data between 1970 and 2022,[9] in order to highlight the main trade offs that the government faces when deciding the optimal inflation and deficit values. The details on this numerical implementation can be found in Appendix D.

As I established in Proposition 6, the prudent government would like to both reduce debt and increase its reputation in order to achieve higher payoffs. However, in this game it is not possible to do both: the prudent government faces a trade off between either generating higher reputation or reducing debt. Given the states (b, ρ), in order to gain reputation the prudent government needs to choose (π, d) that separates it enough from $\left(\pi^{\xi^I}(b, \rho), d^{\xi^I}(b, \rho)\right)$. In particular, $s_I < s_P$ forces the prudent government to choose an inflation level well below what the imprudent government would choose. But this comes with the cost of increasing debt, since lower inflation implies both a higher real interest rate and lower seigniorage.

9 The value of the parameters considered can be found in Table II.

Figure V illustrates the trade off that the prudent government faces, using as current states $b_{High} = 0.8$ and $\rho_{High} = 0.86$. The first panel of this figure presents the payoffs for the government (computed using Equation (2)) as a function of the inflation choice, and setting the deficit choice equal to the prudent government's optimal deficit decision. The center panel presents the value of b' as a function of inflation, while the third panel presents the reputation update ρ'. Given this states, an imprudent government would choose an inflation value $\pi^{\xi^I}(b_{High}, \rho_{High}) = 0.32$. However, the prudent government has two benefits of choosing a lower inflation rate: first, payoffs increase since $s_I < s_P$ and, second, since the optimal inflation choice (green line) is considerably lower than $\pi^{\xi^I}(b_{High}, \rho_{High})$, the prudent government is earning reputation. This choice comes with a cost since future debt is $b' = 0.58$, which would be lower if the prudent government had pooled with the imprudent one (b' would be around 0.35). Then, in equilibrium, if the prudent government wishes to earn reputation, it comes with the cost of generating a debt that is higher than what the imprudent government would have generated.

Figure V: Gains and Losses of Separation.



NOTES: This graph plots the equilibrium functions considering the values presented in Table II. The left panel of this figure presents the expected payoffs for the prudent government considering the optimal deficit decision but allowing inflation to vary. The center panel presents the expected value of future debt b' given the states (b_{High}, ρ_{High}) as a function of inflation. The right panel presents the expected reputation update as a function of inflation.

Figure VI displays the equilibrium policy functions $(\pi^{\xi}(\cdot, \rho), d^{\xi}(\cdot, \rho), b'^{\xi}(\cdot, \rho))$ as a function of current debt for two values of reputation $\rho_{Low} = 0.12$ and $\rho_{High} = 0.88$. The solid lines represent the prudent government's choices for low (green line) and high (blue line) reputation, while the dashed line represent the imprudent government policy functions. As this figure shows, the prudent government has an inflation policy function that is lower than the imprudent government one, since for the prudent government it is more costly to have an elevated inflation rate. However, when government reputation is low, as debt increases the prudent government generates higher inflation than when the reputation is high (approximately 40% higher than when government reputation is high). There are two main reasons of why inflation for the prudent government is considerably higher when it has low reputation: the effect on the real interest rate (and in turn the effect on debt); and the effect on output. In this model, given the current states and taking as given the strategy of the other players, the evolution of debt is given by:

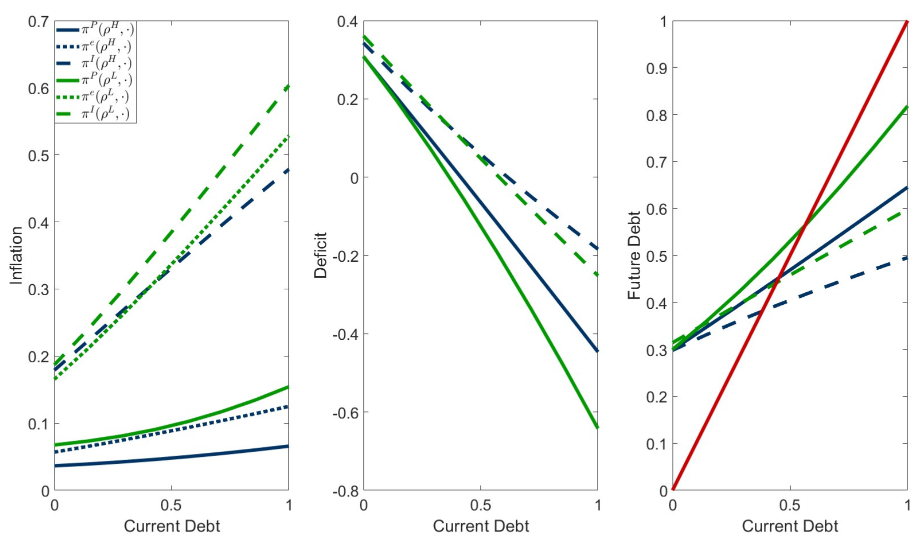
$$b' = d + \frac{(1+r)(1+\sigma_w(b,\rho))b}{1+\pi} - \bar{m}\pi,$$

where (π, d) are values that can be chosen by the prudent government. In equilibrium, whenever ρ is low, $\sigma_w(b,\rho)$ is elevated, since agents belief they are facing the imprudent government with high probability and they know the imprudent government chooses high inflation (that is increasing with b). Hence, if the prudent government were to choose π to be low, the real interest rate that it would need to pay would be higher, which has a negative effect on future debt. Similarly, output in this game is given by:

$$y = \bar{y} + \theta(\pi - \sigma_w(b,\rho)) + d,$$

and when government reputation is low, as the government chooses a low π it is lowering output since $\pi - \sigma_w(b,\rho)$ becomes negative. Both this forces invite the prudent government to have higher inflation when ρ decreases. Notice that this effect is only present in the game with reputational concerns, since in the dynamic game with complete information surprise inflation is zero in equilibrium and, hence, inflation cannot affect neither output nor the real interest rate.

Figure VI: High vs Low Reputation.

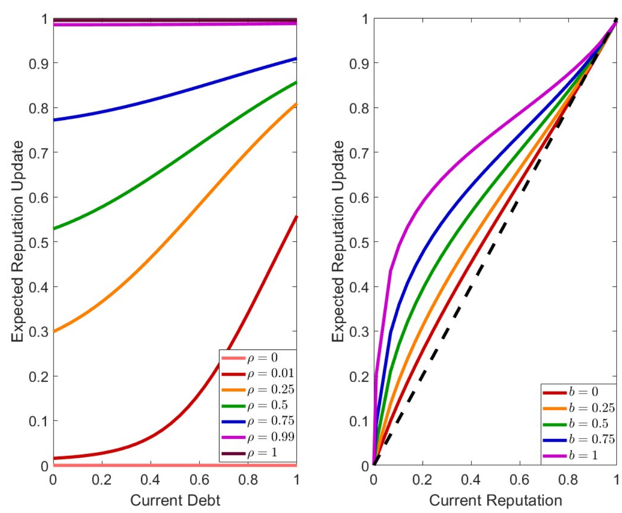


Notes: This graph plots the equilibrium functions considering the values presented in Table II for two values of reputation $\rho_{Low} = 0.12$ and $\rho_{High} = 0.88$. The solid lines represent the policy functions for the prudent government, the dashed lines for the imprudent government, while the dotted lines represent inflation expectations.

As a consequence of generating lower inflation, the prudent government generates a debt policy function that is above the imprudent government debt policy function. Combining this plus what I analyzed in Figure V, in this model the prudent government takes more time to control high debt episodes. The reason behind this are reputation effects: the prudent government has higher payoffs by increasing its reputation over time by having lower inflation and fiscal deficits, even though this might generate higher debt than what an imprudent government would produce. Then, it is more valuable for the prudent government this increase in reputation, than to immediately control debt. Hence, this model generates the prediction that prudent government could generate a time series of inflation and debt that are less correlated, since prudent governments will tend to generate low inflation with more debt. On the other hand, in this model high inflation and high fiscal deficits are evidence suggesting agents are facing an imprudent government.

Given the imperfect monitoring structure, wage setters cannot learn immediately the type of government they are facing. However, the strategy followed by the prudent government in equilibrium takes this into account. Since it is optimal for this government to follow a separating strategy, it is possible for this government to accumulate reputation over time. Figure VII displays the expected reputation update (conditional on the government being prudent) for different values of current debt and reputation. This figure suggests that, in equilibrium, it is more likely for a prudent government with low reputation to increase its reputation than when it already has high reputation. This is due to the fact that when ρ is low, inflation expectations are high and hence the imprudent government chooses an elevated inflation level, as it is displayed in Figure VI. Even with noise, since the prudent government's choices are clearly separated from the imprudent government's decisions, this will likely result in an observed inflation and deficit that are closer to (π^{ξ^P}, d^{ξ^P}), which makes the update ρ' higher than ρ.

Figure VII: Expected Reputation Update Conditional on ξ^P.



NOTES: This graph plots the equilibrium functions considering the values presented in Table II. This figure he expected probability update conditional on the government being of type ξ^P for different values of (b, ρ): $P\left[\rho'(b,\rho) > \rho | \xi^P\right]$. The dashed black line represents the 45 degree line.

The right hand panel of Figure VII also suggests that in this model, a prudent government is more likely to earn reputation as debt increases. Hence, in this framework, "bad times" are ideal for a prudent government to convince wage setters that they are facing a prudent government. In this sense, this model actually predicts that higher debt may not be "bad news" for the government and monetary policy: high debt represents, according to this model, the perfect opportunity to generate low inflation and hence convey to private agents that the government is actually committed to low inflation. On the other hand, in “good times” both the prudent and imprudent government take similar decisions, and hence it is harder for wage setters to discern what they observe into higher prudent government reputation.

Finally, it is a well known result that with imperfect monitoring, reputation effects are necessarily impermanent (Cripps et al. (2004)). The main idea behind this result is that, in equilibrium, in order for agents to be convinced they are facing the imprudent government, even though in reality they are facing the prudent government, the series of shocks $(\epsilon_\pi, \epsilon_d) = (\tilde{\pi} - \pi^{\xi^P}, \tilde{d} - d^{\xi^P})$ would necessarily need to have a different distribution that the one that actually determines shocks. In other words, the prudent government needs to be “very unlucky” for shocks to drive ρ' towards zero in equilibrium, and this “unlucky“ streak has to end eventually, since the distribution of shocks has a mean of zero and therefore $(\tilde{\pi}, \tilde{d})$ are centered around (π^{ξ^P}, d^{ξ^P}). In equilibrium, the prudent government understands this and chooses (π^{ξ^P}, d^{ξ^P}) such that it allows it to eventually separate itself from the imprudent government. Given that all of the equilibrium objects are stationary, one should expect that in this model this results is also true.

Conjecture 2. *Let ϵ_π, ϵ_d be full support distributions and let $\rho_t = P[\xi = \xi^P | h^t, \xi^P]$. Then $\rho_t \to 1$ a.s..*[10]

Conjecture 2 states that, as long as all possible inflation and deficit values can be observed by wage setters when facing either government (the full support condition), if the government is actually prudent, wage setters will eventually learn this. This result in part holds due to the necessary separation that occurs in equilibrium, which makes the distribution of $(\tilde{\pi}, \tilde{d})$ be statistically different if ξ^P or ξ^I are in power. Another property of the model that points towards this conjecture being true is that the reputation process is a sub-martingale, i.e. $E_t[\rho'(b, \rho)] \geq \rho$ for every (b, ρ). This can be observed on the right hand panel of Figure VII. [11]

Full long-run learning is a positive result for the prudent government, in the sense that wages setters will eventually figure out the truth, and in that scenario reputation should no longer be a concern for the prudent government (since $\rho = \rho' = 1$), which means players are back in the dynamic game with no reputation concerns environment, where a prudent government can sustain low inflation with low debt. Of course, there is a critical assumption that needs to hold for this to be true, which is that there is no government turnout, in the sense that the prudent government has to be in power forever.

6 Data Through the Lens of the Model

This section of the paper aims to take my reputations framework to the data of some emerging market economies, in order to see what are the predictions of the model in terms of government reputation and inflation expectations. In general, there is data on inflation and government debt for several countries, however, data on both fiscal deficit and inflation expectations

10 For this result to hold, Cripps et al. (2004) require a second condition which they refer to as *identifiability*. In essence, this condition requires that the distribution of $(\tilde{\pi}, \tilde{d})$ to be different whenever agents face one type or the other. Since in my model, in any equilibrium there is separation at every (b, ρ), this condition holds automatically.

11 If ρ were to be the only state variable, then the sub-martingale property implies full asymptotic learning. However, in my model there is an additional state variable. What I need to show is that, the debt process converges to a steady state, and then, once the economy is in the steady state, and hence debt is fixed, I can then use the sub-martingale property to guarantee full long-run learning.

is either limited or not comparable across countries due to different measurement techniques. Fortunately, and as I describe in this section, my model allows me to infer a time series for government reputation, fiscal deficit, output gap, as well as inflation expectations given a sequence of inflation and debt data. I calibrate each of the countries I present to the input data (inflation and debt), and then use the available data on inflation expectations, deficit, and output to validate the model predictions.

6.1 Data Description

I consider inflation and debt data for four emerging market economies: Mexico, Colombia, Guatemala, and Thailand. I use these countries for several reasons: they have good available data on both variables, they have Central Banks and all of them are currently independent with an inflation target rule, all of these countries issue their own currency, and they have similar methodologies to measure inflation expectations.

In the case of these four countries, I considered the inflation time series that is reported by the World Bank. I also checked that these time series were consistent with the reports made in the web pages of each Central Bank. The advantage of the World Bank data is that they have inflation reported since 1960. Given the scope of my paper and that I want to analyze the correlation between inflation and debt in different time periods, the longer the time series, the better.

The time series for government debt was constructed by me using data on total government debt reported by the World Bank, which I also compared to the public debt reported by the ministry of finance of each country (when possible). In this paper I am focusing on the effect that debt has on government decisions, so I consider reasonable to consider a broad measurement of debt, since the higher total debt is, the larger are the fiscal and monetary burdens that the government has to solve.

In terms of other variables that I use to contrast the model predictions with it, the sources are country specific with the exception of output, which is also reported by the World Bank (I report the source on each country's section). Fiscal deficit time series are reported by either the ministry of finance or the Central Bank of these countries. In three out of four cases (with the exception of Mexico), they start being reported around the year 2000. In terms of inflation expectations, I used Bloomberg data to construct the expected inflation rate for the following twelve months, with the exception of Mexico, for which I considered Banco de Mexico's inflation expectations survey between 2000 and 2022.

Since there are some parameter restrictions that I need to consider in order to guarantee equilibrium existence, and considering that the main focus of the paper is to understand how the correlation between debt, deficit, and inflation varies with government reputation, it is not important for the model to replicate the level of debt, inflation, and other variables involved. Hence, to simplify both the analysis and the calibration, I use a normalization of the inflation and debt data in order for them to be in the $[0,1]$ interval. I consider:

$$b_t^{N,data} = \frac{1}{max(\{b^{data}\} - min(\{b^{data}\}} \left(b_t^{data} - min(\{b^{data}\} \right),$$

$$\pi_t^{N,data} = \frac{1}{max(\{b\pi^{data}\} - min(\{b\pi^{data}\}} \left(\pi_t^{data} - min(\{\pi^{data}\} \right).$$

I also normalize inflation expectations data in order for them to be comparable with the normalized inflation data.

6.2 Taking the Model to the Data

The following exercise presents the model's predictions for government reputation, fiscal deficit, expected inflation, and output gap given data on inflation and government debt. I consider data between 1970 and 2022 for these four countries. In the model, wage setters observe the previous history of play, consistent of $(\tilde{\pi}, \tilde{d}, \tilde{b}')$ for each period in order to generate the current value of government reputation. That is, the value of (b, ρ) in the current period determines both wage setters' inflation expectations and government behavior. Given this, the on-path evolution of debt is then:

$$b'(b,\rho) = \tilde{d}(b,\rho) + \frac{(1+r)(1+\sigma_w(b,\rho))b}{1+\tilde{\pi}(b,\rho)} - \bar{m}\tilde{\pi}(b,\rho)$$

Since I have data on inflation and debt $(\pi_t^{data}, b_t^{data})$, given a set of calibrated parameters and assuming a value for (b_0, ρ_1), I can then use the data time series to construct a model predicted series for inflation expectations $\sigma_w(b_{t-1}^{data}, \rho_t)$; model consistent fiscal deficits:

$$\tilde{d}_t = d_t(b_{t-1}^{data}, \rho_t) = b_t^{data} + \bar{m}\pi_t^{data} - \frac{(1+r)(1+\sigma_w(b_{t-1}^{data}, \rho_t))b_{t-1}^{data}}{1+\pi_t^{data}};$$

output gap:

$$y_t(b_{t-1}^{data}, \rho_t) - \bar{y} = \theta\left(\pi^{data} - \sigma_w(b_{t-1}^{data}, \rho_t)\right) + d_t(b_{t-1}^{data}, \rho_t);$$

and, finally, a time series for reputation with update:

$$\rho_{t+1}(b_{t-1}^{data}, \rho_t) = \frac{\rho_t g_\pi\left(\pi_t^{data} - \pi^{\xi^P}(b_{t-1}^{data}, \rho_t)\right) g_d\left(\tilde{d}_t - d^{\xi^P}(b_{t-1}^{data}, \rho_t)\right)}{\rho_t g_\pi\left(\pi_t^{data} - \pi^{\xi^P}(b_{t-1}^{data}, \rho_t)\right) g_d\left(\tilde{d}_t - d^{\xi^P}(b_{t-1}^{data}, \rho_t)\right) + (1-\rho_t) g_\pi\left(\pi_t^{data} - \pi^{\xi^I}(b_{t-1}^{data}, \rho_t)\right) g_d\left(\tilde{d}_t - d^{\xi^I}(b_{t-1}^{data}, \rho_t)\right)}.$$

6.3 Parameter Calibration and Identification

To calibrate the model, I consider a grid of different parameter values (γ, δ_P, θ, s_P, s_I, $\bar{m}$, $\bar{y}$, r, σ_π, σ_d) and chose the parameter values within the grid that minimized the distance between the model implied inflation expectations and the inflation data. By construction, both the implied debt and inflation generated by the model exactly matches the data, and I do not have a complete time series for fiscal deficits. This is why I decided to use the parameters that make inflation expectations closer to the inflation data.

Both the inflation and debt series are important to identify the key parameters of the model. As discussed in Section 4, as more imprudent a government is, the more likely it is to produce a sequence of elevated debt. Also, the parameter γ governs the dislike that a government has for generating higher debt. Both parameters are then affected by how controlled/elevated is debt in the data. For example, in the countries considered, Colombia and Thailand experienced prolonged episodes of high debt during the sample period; while Guatemala only has had one high debt episode (during the 80s). This is why the model identifies Colombia and Thailand as being the most imprudent countries, as well as having the lowest γ.

On the other hand, the level on both high/low inflation episodes allows the model to identify the values of s_P, s_I. Whenever inflation is low and close to $\bar{\pi}$, it is more likely that wage setters are facing the prudent government. Longer periods of low inflation close to the target indicate a higher s_P. Of the countries analyzed, Mexico is the country that has had inflation closer to the target for the longest period (since 2005), reason for which the model attributes it the highest value of s_P. The value of s_I is identified using the high inflation episodes and its duration, since it is more likely for the wage setters to believe they are facing an imprudent government during these times.

One of the key parameters to be identified is ϵ_π, since it influences the likelihood of facing one government type given high/low inflation episodes. The volatility of the inflation data is what helps to determine this parameter, in particular, how quick does a country shifts between high inflation times to controlled inflation. Colombia is the country that had the most prolonged inflation episodes, and took around 10 years to transition from high to controlled inflation. The model estimates that Colombia has the highest value of ϵ_π within the four countries studied.

The parameter ϵ_d is the most challenging for the model to identify due to the lack of direct deficit data, leading to a noisier estimation. The deficit time series used to identify this parameter is the one that is model consistent, which uses both inflation and debt data. This constructed deficit series could vary due to either changes in inflation, debt, or model inflation expectations. Proof of this is that ϵ_d is quite similar for all four countries, even though each of them have quite different histories of debt and inflation.

6.4 Debt, Inflation, and Government Reputation

In this section I analyze the recent history of debt and inflation through my model's insights for four different emerging markets (Mexico, Colombia, Guatemala, and Thailand). Then, I compare each country's prediction with the available data for inflation expectations and output gap. For the case of Mexico, Colombia, and Guatemala, I also compare the model's predictions about government reputation, inflation, expectations, deficit, and debt with the historical account of each country presented in Kehoe and Nicolini (2021).

6.4.1 Mexico

The calibrated parameter values for Mexico are presented in Table II. According to the model, the prudent version of the Mexican government has a discount factor of $\delta_P = 0.45$ and a parameter $s_P = 80$ that is 16 times larger than the disutility the imprudent government receives if it generates high inflation away from $\bar{\pi} = 3\%$.

Figure VIII displays the model predictions in terms of prudent/imprudent government behavior, fiscal deficit, inflation expectations, and government reputation for Mexico between 1970 and 2022. This country has had two high debt episodes in its history: between 1982-1989 and between 2016-2022. The highest inflation rate in Mexico presented during 1986. Given that during the 80s Mexico produced high inflation with high debt series, as well as high deficits (according to the model), this is consistent with agents facing an imprudent government. This is the reason why government reputation during the 80s was near zero. After 1995, inflation began to become more stable and during 2000-2016 it became a persistent and controlled process (as documented by Ramos-Francia and Torres-Garcia (2005)). Since inflation became

Table II: CALIBRATED PARAMETER VALUES FOR MEXICO.

Parameter	Interpretation	Value
δ_P	Prudent Government Discount Factor	0.45
s_P	Prudent Government Inflation Disutility	80
s_I	Imprudent Government Inflation Disutility	5
θ	Sensitivity of Output to Inflation	0.5
k	Time Inconsistency Parameter	2
γ	Debt Weight	2
$\bar{\pi}$	Inflation Target	3%
r	Interest Rate	5%
$\sigma\pi$	Standard Deviation Inflation Shock	0.15
σd	Standard Deviation Deficit Shock	0.2

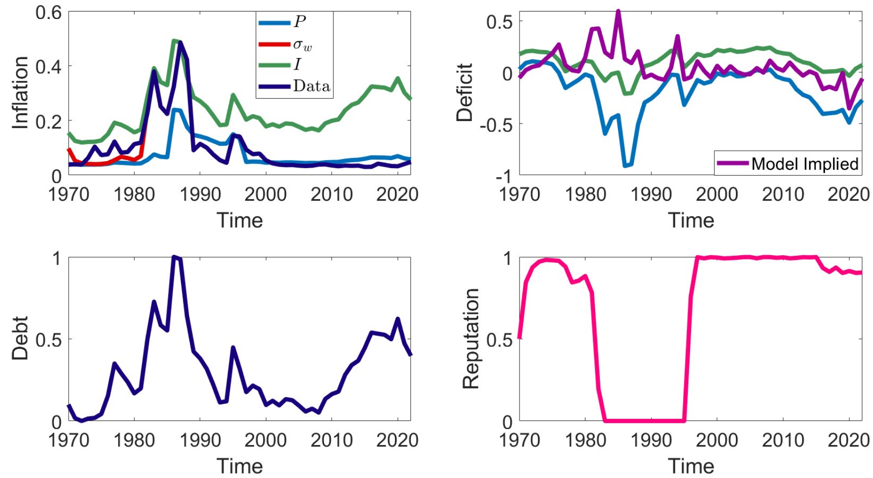
controlled, government reputation increased considerably during 1996-2010. This is consistent with the fact that in Mexico, in 1995 the Central Bank became independent and in 2002 the Central Bank announced it was changing its policy towards an inflation targeting regime. Nevertheless, since 2015 debt began to increase considerably, and inflation rose during 2016-2019, part of the reason why government reputation has still not converged towards 1. In the light of this model, in order for the Mexican government to earn reputation (and hence for agents to believe they are facing a government more committed to low inflation), it should produce an inflation time series that is low and less correlated with the increasing debt. In addition, given the elevated debt, a more prudent government should produce a fiscal deficit time series that is more aggressive in deficit reduction (as shown in blue in the top right panel), however, the model implied deficit series (in purple) is more similar to what an imprudent government would do, which is also the reason why ρ is not closer to 1 between 2015-2022.

The values for government reputation that the model predicts seem to be consistent with the important reforms that took place in Mexico towards controlling inflation and generating credibility for monetary policy. To further validate the model predictions, I now compare the model implied series for inflation expectations and output with the available data, neither which were used to calibrate the model. Figure IX displays the model and data time series between 2000-2022. The model does a good job generating time series that are actually similar to the ones from the data. The correlation between the model and inflation expectations data is 0.87, while the correlation between the model and the output gap data is 0.37. Inflation expectations according to the model are slightly above what we observe in the data, this is due to the fact that debt has been increasing since 2015. In the model, higher debt will be reflected in an increase of inflation expectations, irregardless of the value of government reputation. On the other hand, higher debt translates into lower fiscal deficit in the model, which closes the output gap, but in recent years the Mexican government has actually been generating higher fiscal deficits.

6.4.2 Colombia

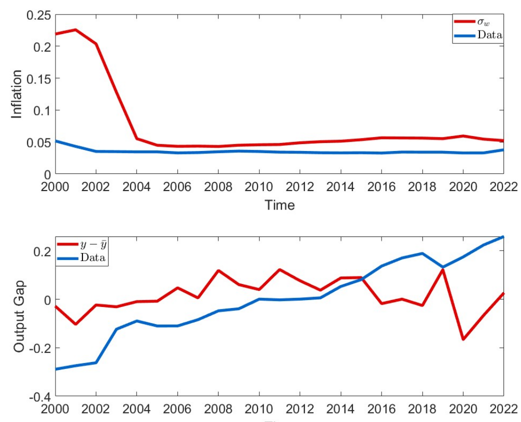
Table III presents the calibrated parameter values in order for the inflation expectations generated by the model to be close to the inflation data from Colombia. Compared to Mexico, the model predicts that the Colombian prudent government has a lower discount factor ($\delta_P = 0.25$), which is why we observe prolonged high debt episodes in Colombia compared to Mexico (especially between 1980-1995).

Figure VIII: MODEL PREDICTIONS FOR MEXICAN DATA.



NOTES: This graph plots the equilibrium functions considering the values presented in Table II. The top left panel presents the model predicted inflation series for the prudent government (blue), imprudent government (green), inflation expectations (red), as well as the inflation data (navy blue). The top right panel displays the model implied deficit sequence for the prudent government (blue), imprudent government (green), as well as the model consistent deficit sequence (purple). The bottom left panel presents the debt data (navy blue). The bottom right panel presents the model implied government reputation (pink).

Figure IX: MEXICO'S INFLATION EXPECTATIONS AND OUTPUT DATA.



NOTES: The red line in both panels displays the model's predictions for inflation expectations and output gap between 2000-2022. The blue line represents the available data on both these variables. Inflation expectations data come from Banco de Mexico's inflation expectations survey, while the output gap series was elaborated by me using the Hodrick-Prescott filter with Mexico's GDP data from the World Bank.

Figure X displays the model's predictions for inflation expectations, fiscal deficit, and government reputation that are consistent with the inflation and debt data in Colombia between 1970-2022. According to the model, government reputation in this country was low during

Table III: CALIBRATED PARAMETER VALUES FOR COLOMBIA.

Parameter	Interpretation	Value
δ_P	Prudent Government Discount Factor	0.25
s_P	Prudent Government Inflation Disutility	80
s_I	Imprudent Government Inflation Disutility	8
θ	Sensitivity of Output to Inflation	1
k	Time Inconsistency Parameter	3
γ	Debt Weight	0.75
$\bar{\pi}$	Inflation Target	3%
r	Interest Rate	5%
$\sigma\pi$	Standard Deviation Inflation Shock	0.3
σd	Standard Deviation Deficit Shock	0.25

1980-2007, which led to high inflation between 1977-1995 as well as an elevated debt level. In Colombia, during the 70s, economic activity was increased due to the "coffee boost" regime, in which the price of coffee (one of the most important commodities exported by Colombia) was high and stable. During the 80s the price of coffee decreased considerably, together with international oil prices, which brought fiscal imbalances and high debt, together with economic stagnation and inflation. It was until 1992 when a new constitution was established in Colombia, when both trade liberalization agreements as well as the independence of Colombia's Central Bank, took place, both reforms helping to reduce debt and inflation. In 2001 Colombia established an inflation targeting regime, and inflation began to decrease towards the 3% target. The model attributes this changes to an earn in government reputation starting at 2007, which was reinforced by the fact that during 2010-2020 debt increased considerably but inflation remained controlled. Nevertheless, since inflation has spiked since 2020 and debt has still continue to grow, this has affected negatively the value of government reputation, which has decreased.

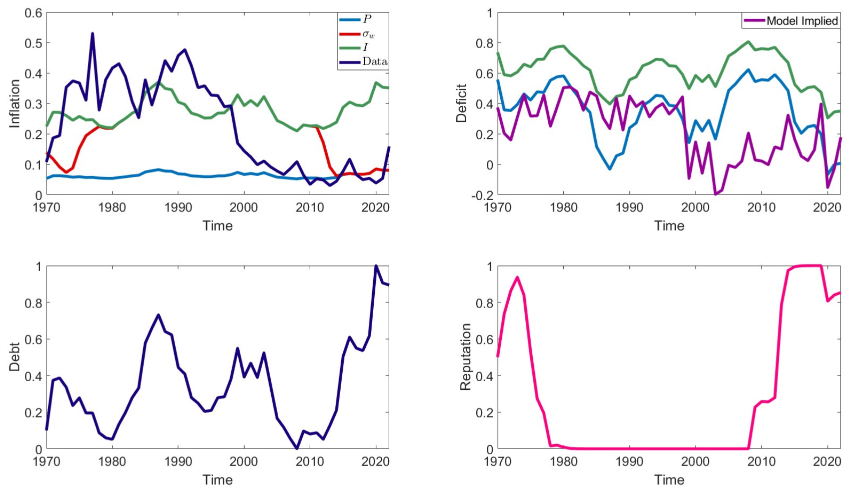
Figure XI presents the comparison between the model's predictions for inflation expectations as well as output with Colombia's data between 2003-2022. The correlation between the model's inflation expectations series and the data is 0.79 while the correlation between the output gap of the model and the data is 0.49. As in the case of Mexico, the model's inflation expectations series is elevated since Colombia's debt has been increasing in recent years, and in the model that is translated into higher inflation expectations irregardless of reputation. On the other hand, the model's output gap is a bit more volatile than the data, since the model implied deficit series is quite volatile (and in equilibrium output is directly affected by deficit fluctuations).

6.4.3 Guatemala

The calibrated parameter values for Guatemala are displayed in Table IV. This are similar values to the ones of Colombia, although the main difference is in the value of δ_P. Unlike Mexico and Colombia, Guatemala only has had in recent years one episode of high debt, between 1980-1990. In the model, governments with higher δ_P care both more about the evolution of debt as well as its reputation, so it is to expect that as δ_P increases, the debt time series generates is more controlled.

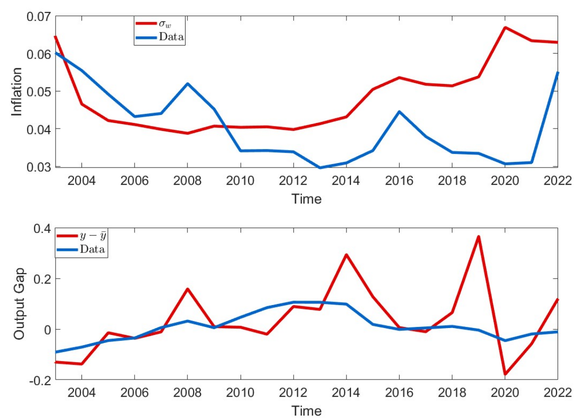
Figure XII displays the model implied time series that are consistent with the inflation and debt data. The model predicts that in order to generate the high inflation with high debt episode

Figure X: MODEL PREDICTIONS FOR COLOMBIAN DATA.



NOTES: This graph plots the equilibrium functions considering the values presented in Table III. The top left panel presents the model predicted inflation series for the prudent government (blue), imprudent government (green), inflation expectations (red), as well as the inflation data (navy blue). The top right panel displays the model implied deficit sequence for the prudent government (blue), imprudent government (green), as well as the model consistent deficit sequence (purple). The bottom left panel presents the debt data (navy blue). The bottom right panel presents the model implied government reputation (pink).

Figure XI: COLOMBIA'S INFLATION EXPECTATIONS AND OUTPUT DATA.



NOTES: The red line in both panels displays the model's predictions for inflation expectations and output gap between 2000-2022. The blue line represents the available data on both these variables. Inflation expectations data come from Banco de Mexico's inflation expectations survey, while the output gap series was elaborated by me using the Hodrick-Prescott filter with Mexico's GDP data from the World Bank.

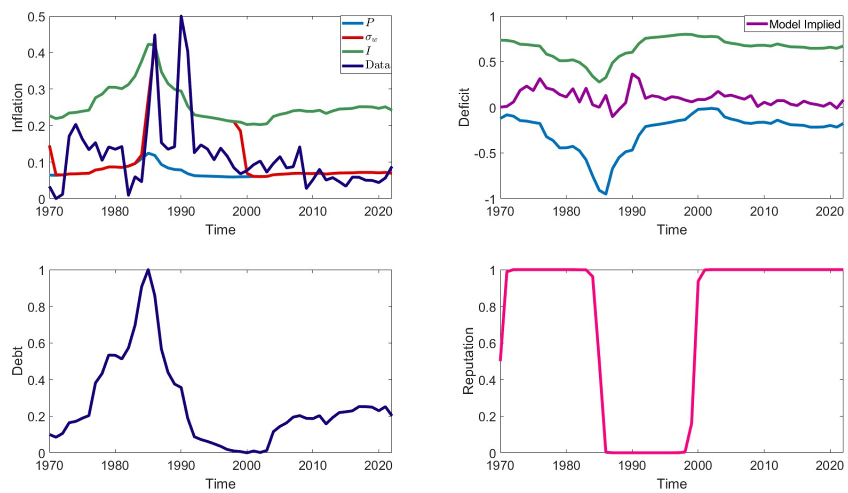
that occurred in Guatemala during the 80s, wage setters had to believe they were facing the imprudent government with high probability. After 1988, debt started to be more controlled, although inflation had a spike 1991. This can only be explained by the model if inflation ex-

Table IV: CALIBRATED PARAMETER VALUES FOR GUATEMALA.

Parameter	Interpretation	Value
δ_P	Prudent Government Discount Factor	0.9
s_P	Prudent Government Inflation Disutility	100
s_I	Imprudent Government Inflation Disutility	15
θ	Sensitivity of Output to Inflation	0.5
k	Time Inconsistency Parameter	3
γ	Debt Weight	2
$\bar{\pi}$	Inflation Target	3%
r	Interest Rate	5%
$\sigma\pi$	Standard Deviation Inflation Shock	0.1
σd	Standard Deviation Deficit Shock	0.2

pectations are high, which is a sign of low government reputation. Guatemala's Central Bank became independent in 1993, and since the early 2000s the Bank has followed an inflation targeting rule of 3%. This has allowed inflation to be controlled and become a more stable process. Since debt has also been controlled and has not come back to the value it had during the 80s, this has contributed to increase Guatemala's government reputation, and bring inflation expectations to a more reduced level.

Figure XII: MODEL PREDICTIONS FOR GUATEMALAN DATA.



NOTES: This graph plots the equilibrium functions considering the values presented in Table IV. The top left panel presents the model predicted inflation series for the prudent government (blue), imprudent government (green), inflation expectations (red), as well as the inflation data (navy blue). The top right panel displays the model implied deficit sequence for the prudent government (blue), imprudent government (green), as well as the model consistent deficit sequence (purple). The bottom left panel presents the debt data (navy blue). The bottom right panel presents the model implied government reputation (pink).

Guatemala measures inflation expectations since 2010, using a survey made to specialists of the private sector. The correlation between my model's predictions and the data on inflation expectations is of 0.82. On the other hand, the correlation between my model implied output gap and the Guatemalan data (which I calculated myself using data from the World Bank) is of 0.67.

6.4.4 Thailand

The calibrated parameters for Thailand are displayed in Table V. Thailand has almost the same parameter values as Guatemala, except for δ_P, which is much lower in Thailand. The reason of this is that, contrary to Guatemala, in this country debt has been elevated for a prolonged period of time (basically since 1990).

Table V: CALIBRATED PARAMETER VALUES FOR THAILAND.

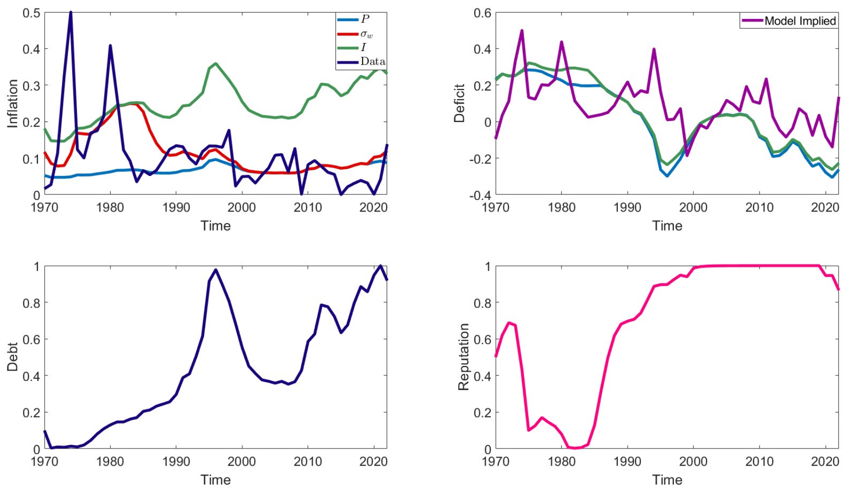
Parameter	Interpretation	Value
δ_P	Prudent Government Discount Factor	0.17
s_P	Prudent Government Inflation Disutility	65
s_I	Imprudent Government Inflation Disutility	5
θ	Sensitivity of Output to Inflation	2
k	Time Inconsistency Parameter	1.5
γ	Debt Weight	1
$\bar{\pi}$	Inflation Target	3%
r	Interest Rate	5%
$\sigma\pi$	Standard Deviation Inflation Shock	0.15
σd	Standard Deviation Deficit Shock	0.25

Figure XIII displays the model's predictions for Thailand between 1970-2022. Thailand, unlike the other countries analyzed before, has the peculiarity of experiencing a high inflation episode in a context of low debt (1975). This is a scenario to which my model cannot fully accommodate. The way the model is constructed allows it to explain high debt with high inflation scenarios, or low inflation with high debt. This type of "escape dynamics" with strong fundamentals, in the sense of Sargent et al. (2009), is a feature of the data my model cannot explain. The best my model can do is to assign a very low reputation, since the imprudent government tends to generate higher inflation. However, the reputation prediction during 80s of my model is not as sharp as in the previous countries, due to the fact that high inflation with low debt is attributed to a very high inflationary shock, an event that for both types has very low probability.

Since the 90s, debt in Thailand has been elevated, even though inflation has been trending downwards to become more stable. This is, in the context of my model, the ideal scenario for government to earn reputation. Nevertheless, since debt has been increasing and during 2020-2022 inflation has also been high, this (as in the case of Mexico and Colombia) has translated in a slight decrease of government reputation.

Thailand measures inflation expectations since 2005. The 12 months ahead inflation expectations time series was constructed by me using Bloomberg and Consensus data. The correlation between my model's predictions and the data on Thailand's inflation expectations is of 0.76. On the other hand, the correlation between my model implied output gap and the data (which I calculated myself using data from the World Bank) is of 0.59.

Figure XIII: MODEL PREDICTIONS FOR THAILAND DATA.



NOTES: This graph plots the equilibrium functions considering the values presented in Table IV. The top left panel presents the model predicted inflation series for the prudent government (blue), imprudent government (green), inflation expectations (red), as well as the inflation data (navy blue). The top right panel displays the model implied deficit sequence for the prudent government (blue), imprudent government (green), as well as the model consistent deficit sequence (purple). The bottom left panel presents the debt data (navy blue). The bottom right panel presents the model implied government reputation (pink).

7 Conclusions

This paper presents a dynamic game with incomplete information, where private agents form beliefs about the type of government they are dealing with. The evolution of these beliefs, along with public debt levels, play a crucial role in determining inflation, deficits, and inflation expectations. The model predicts that as public debt increases, inflation tends to rise. However, when the government's reputation is strong (indicating a high commitment to low inflation) the impact of debt on both inflation and inflation expectations diminishes. Conversely, when the government's reputation is weak, increases in debt are more likely to lead to significant inflation spikes. Thus, the model suggests a high correlation between public debt, inflation, and inflation expectations when government reputation is low, and a weaker correlation when reputation is high.

To validate the model, I apply it to data from four emerging market economies, examining the interplay between government reputation, inflation expectations, inflation, and public debt over time. The results underscore the importance of maintaining low inflation to bolster government reputation. These findings highlight the challenges faced by many economies over the past decade, where rising debt levels have coincided with higher inflation, leading to a slight decline in the public's confidence about the government's commitment to control prices.

Historically, episodes of high public debt have been viewed as "bad news" for governments, often associated with subsequent inflationary pressures. However, this model offers a nuanced perspective: a high debt scenario can be an opportunity for a government committed to low inflation to reinforce its credibility by producing a sequence of low, debt-unrelated inflation outcome.

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A Static Nash Equilibrium Proofs

This section discusses the proof that characterizes the static Nash equilibrium of the game described in Section 3.3. This is a static one-shot game in which the government chooses both inflation and deficit levels, while the wage setters decide their inflation expectations. Both agents take as given the current debt level in the economy, b. In this appendix, I first prove that the best-response of each player is unique, then I show that there is a unique equilibrium of this game, and I characterize how equilibrium behavior reacts as b changes.

Before delving into these proofs, I first present two results that I use not only in this section, but in further parts of the paper as well.

Lemma 1. *1. Let $g(x, y)$ be a strictly concave function in (x, y) and $h(x)$ be a strictly concave function. Then $f(x, y) = g(x, y) + h(x)$ is a strictly concave function.*

2. Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable strictly concave function and $h : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $h' \neq 0$. Then $f = g \circ h$ is strictly concave if either g is strictly increasing and h is concave or g is strictly decreasing and h is convex.

Proof. 1. Let $(x_0, y_0) \neq (x_1, y_1)$ and $\lambda \in (0, 1)$. Then:

$$f(\lambda x_0 + (1-\lambda)x_1, \lambda y_0 + (1-\lambda)y_1) = g(\lambda x_0 + (1-\lambda)x_1, \lambda y_0 + (1-\lambda)y_1) + h(\lambda x_0 + (1-\lambda)x_1).$$

Then, since g, h are strictly concave functions then $g(\lambda x_0 + (1-\lambda)x_1, \lambda y_0 + (1-\lambda)y_1) > \lambda g(x_0, y_0) + (1-\lambda)g(x_1, y_1)$ and $h(\lambda x_0 + (1-\lambda)x_1) > \lambda h(x_0) + (1-\lambda)h(x_1)$, which implies that:

$$f(\lambda x_0 + (1-\lambda)x_1, \lambda y_0 + (1-\lambda)y_1) >$$
$$\lambda g(x_0, y_0) + (1-\lambda)g(x_1, y_1) + \lambda h(x_0) + (1-\lambda)h(x_1) = \lambda f(x_0, y_0) + (1-\lambda)f(x_1, y_1) =$$
$$\lambda f(x_0, y_0) + (1-\lambda)f(x_1, y_1).$$

2. Let $f(x) = g(h(x))$. Then:

$$f''(x) = \left[h'(x)\right]^2 g''(x) + g''(x)f'(x).$$

Notice that the term $[h'(x)]^2 g''(x)$ is always negative since g is strictly concave and $h' \neq 0$. Then, $f''(x) < 0$ for all x if the second term is either zero or negative. This is guaranteed to happen whenever g is strictly increasing and h is concave or g is strictly decreasing and h is convex.

□

Proposition 8. *Taking as given the strategy of the other player, each player has a unique best response.*

Proof. Taking as given $\pi(b)$, the wage setters best-response is the solution to:

$$\max_{\pi^e} -(\pi^e - \pi(b))^2.$$

This is a strictly concave function in π^e and hence there is a unique solution for which the first order conditions are necessary and sufficient. Now, taking as given π^e, the government's problem is:

$$\max_{\pi,d} -((1-k)\bar{y}+\theta(\pi-\pi^e(b))+d)^2-\gamma\left(d+\frac{(1+r)(1+\pi^e(b))b}{1+\pi}-\bar{y}\pi\right)^2-s\left(\pi-\bar{m}\right)^2.$$

Again, there is a unique best-reply if it is the case that the government's objective is a strictly concave function of (π, d). Let us think of the objective function as $f(\pi,d) = g(\pi,d) + h(\pi,d) + q(\pi)$. Then, by Lemma 1, f will be strictly concave if g, h are strictly concave in (π, d) and q is strictly concave in π. Once again, now using the second part of Lemma 1, g, h are indeed strictly concave functions, since they both are a composition of strictly concave quadratic function with a linear function (case of g) and a strictly concave function (case of h). □

A.1 Proof of Proposition 1.

1. $\pi^\star(b)$ is an increasing function of b.

   Proof. The first order condition of the wage setters' problem implies that $\pi^e = \pi$. On the other hand, the optimal solution of the government's problem given π^e is the (implicit) solution of:

$$\theta((1-k)\bar{y}+\theta(\pi-\pi^e)+d)+s(\pi-\bar{\pi})-\gamma\left(\frac{(1+r)(1+\pi^e)b}{(1+\pi)^2}-\bar{m}\right)\left(d+\frac{(1+r)(1+\pi^e)b}{1+\pi}-\bar{m}\pi\right)=0, \tag{A4}$$

$$(1-k)\bar{y}+\theta(\pi-\pi^e)+d+\gamma\left(d+\frac{(1+r)(1+\pi^e)b}{1+\pi}-\bar{m}\pi\right)=0. \tag{A5}$$

   Substituting the equilibrium condition $\pi = \pi^e$ then, using Equation (A5) we get that:

$$d=\frac{(k-1)\bar{y}-(1+r)b+\bar{m}\pi}{1+\gamma}. \tag{A6}$$

   Substituting this in Equation (A4) we get that:

$$F(\pi,b)=$$

$$\theta(k-1)+\theta\left[\frac{(k-1)\bar{y}-(1+r)b+\bar{m}\pi}{1+\gamma}\right]+s(\pi-\bar{\pi})$$

$$-\gamma\left(\frac{1+r}{1+\pi}+\bar{m}\right)\left(\frac{(k-1)\bar{y}-(1+r)b+\bar{m}\pi}{1+\gamma}+(1+r)b-\bar{m}\pi\right)=0.$$

   I will use the Implicit Function Theorem to characterize the solution $\pi(b)$. The derivatives of F are:

$$\frac{\partial F}{\partial \pi}=\frac{\theta\bar{y}}{1+\gamma}+s+\frac{(1+r)\gamma\bar{m}^2}{(1+\pi)(1+\gamma)}+\frac{(1+r)((k-1)\bar{y}+\gamma(1+r)b+\gamma\bar{m}\pi}{(1+\pi)^2(1+\gamma)}>0,$$

$$\frac{\partial F}{\partial b}=-\frac{(1+r)\theta}{1+\gamma}-\gamma^2\left(\frac{1+r}{1+\pi}\right)\frac{1+r}{1+\gamma}<0.$$

Then, by the implicit function theorem:

$$\frac{d\pi}{db} = -\frac{\frac{\partial F}{\partial b}}{\frac{\partial F}{\partial \pi}} > 0,$$

implying that $\pi(b)$ is an increasing function of b. Finally, one can show that $\frac{d\pi}{db} < 1$, which will be used in the following proof. □

2. $d^\star(b)$ is a decreasing function of b.

   Proof. Using Equation (A6), it is the case that:

$$\frac{dd}{db} = \frac{1}{1+\gamma}\left[-(1+r) + \bar{m}\frac{d\pi}{db}\right].$$

   Since $\bar{m} \leq 1 < 1 + r$ and $\frac{d\pi}{db} < 1$, then the derivative of d with respect to b must be negative. □

3. Wage setters' payoffs in equilibrium are zero, while the government's payoffs are decreasing in b.

   Proof. Since in equilibrium $\pi^e = \pi$ (no surprise inflation), then wage setters have a payoff of zero.

   Now, the government solves the following (relaxed) problem to obtain its best replies of inflation and deficit:

$$V^{NE}(b) = \max_{\pi,d} \; -(y - k\bar{y})^2 - s(\pi - \pi^e)^2 - (b')^2 \text{ subject to}$$

$$\bar{y} + \theta(\pi - \pi^e(b)) + d \geq y,$$

$$b' \geq d + \frac{(1+r)(1+\pi^e(b))b}{1+\pi} - \bar{m}\pi,$$

$$0 \leq b' \leq \bar{b}.$$

   This relaxed version attains the same optimal value as the problem with equalities (since the government likes generating higher output and lower debt). Let $\lambda_1, \lambda_2 > 0$ be the Lagrange multipliers associated with the first and second restriction (respectively). Then, the envelope theorem implies:

$$\frac{\partial V^{NE}}{\partial b}(b) = \frac{\partial \mathcal{L}}{\partial b} = -\lambda_1 \pi^{e'}(b) - \lambda_2 \left[\frac{(1+r)(1+\pi^e(b))}{1+\pi(b)} + \frac{(1+r)\pi^{e'}(b)b}{1+\pi(b)}\right].$$

   Since, in equilibrium $\pi^e(b) = \pi(b)$ and $\pi(b)$ is increasing, then:

$$\frac{\partial V^{NE}}{\partial b}(b) = \frac{\partial \mathcal{L}}{\partial b} = -\lambda_1 \pi^{e'}(b) - \lambda_2 \left[1 + r + \frac{(1+r)\pi^{e'}(b)b}{1+\pi(b)}\right] < 0.$$

□

B Dynamic Game Proofs

In order to simplify part of the existence and characterization proofs in both the dynamic and reputation games, I impose the following assumptions on the domain for inflation, deficit, as well as a constraint on the parameter space.

Assumption 3. The domain for inflation is $\pi \in [0,1]$, the domain for fiscal deficit is $d \in [-1,1]$, and a restriction for the model parameters is:

$$(1-k)\bar{y} + \theta \leq 0.$$

The economic intuition behind this parameter restriction is the following: it is not enough to use inflation to have output above $k\bar{y}$. In the best-case scenario for the government, inflation expectations are equal to zero, and hence the best inflation rate to boost output would be $\pi = 1$, which implies that $y - k\bar{y} = (1-k)\bar{y} + \theta$ whenever $d = 0$. This assumption states that, under this scenario, the economy still needs a positive deficit to increase production above $\bar{y}k$.

In this section, I present the proofs for the dynamic game imposing an additional assumption, which is just for simplicity. Then, I explain how to modify the proofs in order to relax this assumption. I assume that I am modelling a **small open economy** (SOE), in the sense that in this economy, the real interest rate is exogenously given. Hence, the evolution of debt becomes:

$$b' = d + (1+r)b - \bar{m}\pi.$$

Notice that, in the baseline model presented in Section 4, on the equilibrium path debt has the same dynamics as in a SOE. However off-path behavior differs since inflation may not be equal to expected inflation. Now I present some definitions and lemmas that will be useful for the main proofs.

In what follows, it is important to highlight that the best reply of the government takes as given the strategy of wage setters. To make explicit that the strategy of wage setters is taken as given, I denote $F(\cdot|\sigma_w)$ all the objects of the governments problem, which are taking into account the strategy of wage setters.

Definition. Let $f(\pi, d, b|\sigma_w)$, the flow-payoffs for the government, be defined as:

$$f(\pi, d, b|\sigma_w) = -\left((1-k)\bar{y} + \theta(\pi - \sigma_w(b)) + d\right)^2 - s(\pi - \bar{\pi})^2 - \gamma\left(d + (1+r)b - \bar{m}\pi\right)^2.$$

Notice that this function already includes the two restriction on y, b' that the government considers in its optimization problem.

Lemma 2. *Let σ_w be a strictly convex function. Then, under Assumption 3, f is a strictly concave function in (π, d, b).*

Proof. Let us first consider $h(\pi, d, b) = -\left((1-k)\bar{y} + \theta(\pi - \sigma_w(b)) + d\right)^2$. I will show that this function is strictly concave as long as σ_w is strictly convex. The Hessian matrix of this function, which exists since σ_w is differentiable, is:

$$H(\pi, d, b) = \begin{pmatrix} -2\theta^2 & -2\theta & 2\theta^2\sigma_w'(b) \\ -2\theta & -2 & 2\theta\sigma_w'(b) \\ 2\theta^2\sigma_w'(b) & 2\theta\sigma_w'(b) & -2\theta^2(\sigma_w'(b))^2 + 2\theta\left((1-k)\bar{y} + \theta(\pi - \sigma_w(b)) + d\right)\sigma_w''(b) \end{pmatrix}$$

In order for h to be strictly concave, the three elements on H's diagonal must be negative, the first and third leading principal minors of h should be negative, and the second leading principal minor should be positive. The first leading principal minor is $-2\theta^2$, which is negative as long as $\theta > 0$. The second leading principal minor is $8\theta^2 > 0$, while the third leading principal minor is negative as long as as $\pi, \sigma_w \in [0,1]$, $d \in [-1,1]$, $\sigma_w(b)'' > 0$, and the parameter restriction stated in Assumption 3 is satisfied. In terms of the elements in the diagonal, clearly $H_{11} < 0, H_{22} < 0$. In order for $H_{33} < 0$, σ_w must be strictly convex and $(1-k)\bar{y} + \theta(\pi - \sigma(b)) + d < 0$ which is guaranteed to happen as long as as $\pi, \sigma_w \in [0,1]$, $d \in [-1,1]$, and the parameter restriction stated in Assumption 3 is satisfied. Since this Hessian is negative definite, h is strictly concave.

Now, let us consider $g(\pi, d, b) = -\gamma\left(d + (1+r)b - \bar{m}\pi\right)^2$. This is a composition of a linear function of (π, d, b) with a strictly concave function. Hence, it is strictly concave. Finally, the function $q(\pi, d, b) = -s(\pi - \bar{\pi})^2$ is concave as long as $s > 0$. Then, $f = h + g + q$ is strictly concave. □

Definition. The set of optimal choices for the government for each $b \in \mathcal{D}$, given the wage setters' strategy σ_w, is defined as:

$$\Gamma(b|\sigma_w) = \left\{ (\hat{\pi}, \hat{d}) \,\middle|\, V(b|\sigma_w) = \ (1-\delta) f(\hat{\pi}, \hat{d}, b|\sigma_w) + \delta V\left(d + (1+r)b - \bar{m}\pi |\, \sigma_w\right) \right\}.$$

Lemma 3. *For each σ_w, there is a unique solution to the government's recursive problem. Moreover, $\Gamma(b|\sigma_w)$ is a singleton, which implies that there is a unique optimal strategy for the government. Also, this strategy is continuous and differentiable in b.*

Proof. Let us consider the functional T_{σ_w} defined by:

$$T_{\sigma_w}(V)\,(b|\sigma_w) = \max_{(\pi,d)\in\mathcal{D}^2} \ (1-\delta) f(\pi, d, b|\sigma_w) + \delta V\left(d + (1+r)b - \bar{m}\pi |\, \sigma_w\right).$$

Using the same arguments presented in Stokey et al. (1989), we can show that T_{σ_w} satisfies Blackwell's sufficient condition to be a contraction mapping, which implies that this operator has a unique fixed point $V^\star(\cdot|\sigma_w)$. Moreover, since $f(\pi, d, b|\sigma_w)$ is strictly concave and the domain set for (π, d, b) is convex, then $\Gamma(b|\sigma_w)$ must be a singleton for every b as well as a continuous mapping. For more details, see Stokey et al. (1989). □

Proof of Theorem 1.

The proof of this theorem will be based on two classic results in functional analysis: the Schauder Fixed-Point Theorem, and the Arzelà-Ascoli Theorem. For completeness, I first state these theorems first as well as a couple of definitions that are relevant for these theorems.

Definition. Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of continuous functions with domain $I = [a, b]$. We say that:

1. $\{f_n\}_{n\in\mathbb{N}}$ is uniformly bounded if there exists $M > 0$ such that $|f_n(x)| \leq M$ for all $n \in \mathbb{N}$ and all $x \in I$.

2. $\{f_n\}_{n\in\mathbb{N}}$ is equicontinuous if for every $\epsilon > 0$ there exists $\delta > 0$ such that $|f_n(x) - f_n(y)| < \epsilon$ for all $n \in \mathbb{N}$ and all $x, y \in I$ such that $|x - y| < \delta$.

Theorem (Arzelà-Ascoli Theorem). *Let $\{f_n\}_{n\in\mathbb{N}}$ be a sequence of continuous functions with domain $I = [a,b]$. Then, $\{f_n\}_{n\in\mathbb{N}}$ has a uniformly convergent subsequence if and only if $\{f_n\}_{n\in\mathbb{N}}$ is uniformly bounded and equicontinuous.*

Theorem (Schauder Fixed-Point Theorem). *Let $(X, ||\cdot||)$ be a Banach space and let $F \subseteq X$ be a non-empty, compact, and convex set. Let $T : F \to F$ be a continuous mapping. Then, T has a fixed point, that is, there exists $x^\star \in F$ such that $T(x^\star) = x^\star$.*

With this in mind, let me present the proof of Theorem 1. Let Σ_w be the set of twice-differentiable and strictly concave functions such that $|\sigma_w'(b)| \leq M$ for all $\sigma_w \in \Sigma_w$, all $b \in \mathcal{D}$, and some (big) $M > 0$. This means Σ_w is the set of differentiable strictly concave functions with uniformly bounded first derivatives.

This set is non-empty, since the function $\sigma_w(b) = b^2 \in \Sigma_w$. Also, Σ_w is convex, since for any $\sigma_w^1, \sigma_w^2 \in \Sigma_w$, and any $\lambda \in [0,1]$, we have that:

$$\left|\left(\lambda\sigma_w^1(b) + (1-\lambda)\sigma_w^2(b)\right)'\right| \leq \lambda\left|\sigma_w^{1'}(b)\right| + (1-\lambda)\left|\sigma_w^{2'}(b)\right| \leq \lambda M + (1-\lambda)M = M,$$

which implies that $\lambda\sigma_w^1(b) + (1-\lambda)\sigma_w^2(b) \in \Sigma_w$. Finally, let $\{\sigma_w^n\}$ be a sequence of functions such that $\sigma_w^n \in \Sigma_w$ for all $n \in \mathbb{N}$. Let

$$K = \sup_{n\in\mathbb{N}} \max_{b\in\mathcal{D}} \left|\sigma_w^{n'}(b)\right|,$$

which we know is well defined and finite due to $\sigma_w^{n'}$ being continuous (since it is differentiable) and the derivative of all the functions in this sequence is bounded by M. Let $\epsilon > 0$ and consider $\delta = \epsilon/2K$. Consider any $b, b' \in I$ such that $|b - b'| < \delta$. Then, by the mean value theorem:

$$\left|\sigma_w^n(b) - \sigma_w^n(b')\right| = |\sigma_w^{n'}(\xi)||b - b'| \leq K|b - b'| < \frac{\epsilon}{2} < \epsilon,$$

which means that σ_w^n is uniformly equicontinuous. Then, by the Arzelà-Ascoli Theorem, this sequence must have a convergent sub-sequence. Since this happens for an arbitrary sequence in Σ_w, this set is compact.

The next step is to show that the mapping from $\sigma_w \in \Sigma_w$ to $\pi(\cdot|\sigma_w)$ is such that: (1) $\pi(\cdot|\sigma_w) \in \Sigma_w$; (2) is continuous. In order to prove that $\pi(\cdot|\sigma_w) \in \Sigma_w$, I need to show that $\pi(\cdot|\sigma_w)$ is strictly convex and has a first derivative bounded by $M > 0$. Fix $\sigma_w \in \Sigma_w$ and consider the government's problem:

$$V\left(b|\sigma_w\right) = \max_{\pi,d} \ (1-\delta)f(\pi, d, b|\sigma_w) + \delta V\left(g(\pi, d, b)|\,\sigma_w\right),$$

where $g(\pi, d, b) = g_1\pi + g_2 d + g_3 b$ is a linear function. This problem has a unique solution (due to the strict concavity of f), which characterized by the following first order conditions:

$$(1-\delta)f_\pi + \delta g_1 V'(g(\pi, d, b)) = 0$$

$$(1-\delta)f_d + \delta g_2 V'(g(\pi, d, b)) = 0$$

Since f, V are strictly concave and g is linear, I can use the implicit function theorem to characterize $\partial\pi/\partial b$. This theorem implies that:

$$\frac{d\pi}{db} = -\frac{f_{\pi b} + \delta g_1 g_3 V''}{f_{\pi\pi} - \delta g_1^2 V''}$$

Since $V'' < 0, g_1 < 0, g_3 > 0$, then:

$$\frac{d\pi}{db} = -\frac{f_{\pi b} + \delta g_1 g_3 V''}{f_{\pi\pi} - \delta g_1^2 V''} \leq -\frac{f_{\pi b}}{f_{\pi\pi}}$$

Using the definition of f and the fact that $\theta, \gamma, r, s, \bar{m} > 0$, then:

$$\frac{d\pi}{db}(b) \leq \frac{\theta^2 \sigma_w'(b) - \gamma\bar{m}(1+r)}{\theta^2 + s + \gamma\bar{m}^2} \leq \sigma_w'(b) \leq M,$$

implying that $\pi(\cdot|\sigma)$ has a first derivative that is uniformly bounded by M. Now, the second derivative of this function is given by

$$\frac{d^2\pi}{db^2} =$$

$$\frac{f_{\pi bb}\left(f_{\pi\pi} + \delta g_1^2 V''\right) + \delta g_1 V''' g_3 f_{\pi\pi}\left(g_1\pi' + g_2\right)}{\left(f_{\pi\pi} + \delta g_1^2 V''\right)^2}$$

with $f_{\pi bb} < 0, f_{\pi\pi} < 0, g_1 > 0, g_2 > 0, g_3 < 0, V'' < 0, V''' > 0$.[12] Hence, this derivative is strictly positive and, hence, $\pi(\cdot|\sigma_w)$ is strictly concave. In conclusion, $\pi(\cdot|\sigma_w) \in \Sigma_w$.

Now, the space of continuous functions $\mathcal{C}(\mathcal{D})$ is a Banach space with the supremum norm. Hence, viewing σ_w as a parameter of $V(\cdot|\sigma_w)$ (which lives in an infinitely dimensional space), it follows from the Maximum theorem that the mapping $\pi : \Sigma_w \to \Sigma_w$ is continuous. In conclusion, due to the Schauder fixed point theorem, this mapping has to have a fixed point, which is an equilibrium of the dynamic game.

Proof of Proposition 3

The first thing to notice is that, on the equilibrium path, in both the SOE model and the full dynamic model presented in the paper, the evolution of debt is the same. Hence, the qualitative characteristics of the SOE model will be the same as in the model in which the real interest rate is determined endogenously. The following proofs are written considering the SOE model.

1. V is a continuous, decreasing, strictly concave, and differentiable function of $b \in [0, \bar{b}]$.

   Proof. Continuity, strict concavity, and differentiability of V follows from the corollaries of the Maximum Theorem discussed in Stokey et al. (1989). These results follow from the properties of the function f and the fact we are assuming σ_w is strictly convex and differentiable.

   The (relaxed) problem that the government solves is given by:

$$V(b) = \max_{\pi, d, b'} \; (1-\delta)\left[-(y - k\bar{y})^2 - s(\pi - \bar{\pi})^2 - \gamma(b')^2\right] + \delta V(b') \quad \text{subject to}$$

12 The sign of V''' was determined following the analysis of Gerswhin (1969).

$$\bar{y} + \theta(\pi - \sigma_w(b)) + d \geq y,$$

$$b' \geq d + \frac{(1+r)(1+\sigma_w(b))b}{1+\pi} - \bar{m}\pi,$$

$$0 \leq b' \leq \bar{b}.$$

Let $\lambda_1, \lambda_2 > 0$ be the Lagrange multipliers associated to output and debt restrictions (respectively). Then, due to the envelope theorem:

$$\frac{\partial V}{\partial b}(b) = -\lambda_1 \sigma_w'(b) - \lambda_2 \left[(1+r) + \frac{(1+r)\sigma_w'(b)b}{1+\pi(b)} \right] < 0,$$

and hence, $V(\cdot)$ is decreasing in $b \in [0, \bar{b}]$.[13]

□

2. Wage setters' payoffs are zero for all $b \in [0, \bar{b}]$.

   Proof. This is a consequence of no surprise inflation in equilibrium. □

3. $\sigma_w^\star$ is an increasing function of $b \in [0, \bar{b}]$.

   Proof. Given $\sigma_w \in \Sigma_w$, the first-order conditions, which are sufficient and necessary for the government's problem, are (defining $\tilde{y} = (1-k)\bar{y} + \theta(\pi - \sigma_w(b)) + d$):

$$-2\theta\tilde{y} - 2s(\pi - \bar{\pi}) + 2\gamma\bar{m}(d + (1+r)b - \bar{m}\pi) - \delta\bar{m}V'\left(d + (1+r)b - \bar{m}\pi\right) = 0,$$

$$-2\tilde{y} - 2\gamma(d + (1+r)b - \bar{m}\pi) + \delta V'\left(d + (1+r)b - \bar{m}\pi\right) = 0.$$

   In equilibrium, $\sigma_w(b) = \pi(b)$ and hence this FOCs become:

$$-2\theta((1-k)\bar{y}+d) - 2s(\sigma_w(b) - \bar{\pi}) + 2\gamma\bar{m}(d + (1+r)b - \bar{m}\sigma_w(b)) - \delta\bar{m}V'\left(d + (1+r)b - \bar{m}\sigma_w(b)\right) = 0,$$

$$-2((1-k)\bar{y} + d) - 2\gamma(d + (1+r)b - \bar{m}\sigma_w(b)) + \delta V'\left(d + (1+r)b - \bar{m}\sigma_w(b)\right) = 0. \quad \text{(B7)}$$

   Manipulating these equations, we can collapse them into:

$$(\theta + \bar{m})\left((1-k) + d\right) + s(\sigma_w(b) - \bar{m}) = 0.$$

   Then, solving for the deficit from this equation, we get that:

$$d(b) = -(1-k)\bar{y} - \frac{s}{\theta + \bar{m}}(\sigma_w(b) - \bar{\pi}).$$

   Plugging this into (B7), we get:

$$\delta V'\left(d(b) + (1+r)b - \bar{m}\sigma_w(b)\right) = 2((1-k)\bar{y} + d(b) + 2\gamma(d(b) + (1+r)b - \bar{m}\sigma_w(b)).$$

13 I am using the fact that $\sigma_w(\cdot)$ is an increasing function of debt, fact that is proven independently of V being decreasing in statement 3 of this proof.

Differentiating with respect to b, we get:

$$\delta V''(b') \left[1 + r - \sigma_w'(b)\bar{m} - \frac{s}{\theta + \bar{m}} \sigma_w'(b) \right] = 2(1+r) - \frac{2s}{\theta + \bar{m}} \sigma_w'(b) - \frac{2\gamma s}{\theta + \bar{m}} \sigma_w'(b) - 2\gamma \bar{m} \sigma_w'(b).$$

Suppose $\sigma_w'(b) \leq 0$. Since $V''(b) < 0$, then the left-hand side of the previous equation is negative. However, the right-hand side is positive, which is a contradiction. Hence, $\sigma_w'(b) > 0$.

□

4. $\pi^\star(b)$ is a continuous, increasing, and differentiable function of $b \in [0, \bar{b}]$.

   Proof. This is a direct consequence of the following equilibrium property: $\sigma_w^\star(b) = \pi^\star(b)$ for all $b \in \mathcal{D}$. □

5. $d^\star(b)$ is a continuous, concave, and differentiable function of $b \in [0, \bar{b}]$.

   Proof. In equilibrium, the optimal deficit decision satisfies:

$$d(b) = -(1-k)\bar{y} - \frac{s}{\theta + \bar{m}} (\sigma_w^\star(b) - \bar{\pi}).$$

Since the optimal expected inflation is increasing in b, then the deficit must be decreasing. Furthermore since σ_w is convex, then d is concave.

□

C Reputation Game Proofs

C.1 Proof of Theorem 2

As in the dynamic game, I restrict attention to analyze Markov equilibria in which the equilibrium wage setters' strategy $\sigma_w(b, \rho)$ is strictly convex. This assumption, coupled with Assumption 3, guarantees that the government's flow-payoff function is strictly concave, and hence the first order conditions of the government's Bellman problem are necessary and sufficient to pin down best replies.

In addition, I require for σ_w to be continuous and have uniformly bounded first derivatives. Since now this strategy is a function of (b, ρ) this means that there exists $M_1, M_2 > 0$ such that:

$$\frac{\partial \sigma_w}{\partial b}(b, \rho) \leq M_1 \quad \frac{\partial \sigma_w}{\partial \rho}(b, \rho) \leq M_2,$$

for every $(b, \rho) \in [0, \bar{b}] \times [0, 1]$. Let Σ_w be the set of continuous functions with domain $[0, \bar{b}] \times [0, 1]$ that are strictly convex and have uniformly bounded first derivatives. This set is non-empty and convex.

Let $(b_1, \rho_1) \neq (b_2, \rho_2)$ be in the domain of these functions. The two-dimensional version of the mean value theorem implies that there exists $(\tilde{b}, \tilde{\rho})$ in the line segment between these points such that:

$$\sigma_w(b_1, \rho_1) - \sigma_w(b_2, \rho_2) = \frac{\partial \sigma_w}{\partial b}(\tilde{b}, \tilde{\rho})(b_2 - b_1) + \frac{\partial \sigma_w}{\partial \rho}(\tilde{b}, \tilde{\rho})(\rho_2 - \rho_1).$$

Let $\{\sigma_w^n\}$ be a sequence of functions such that $\sigma_w^n \in \Sigma_w$ and let $\epsilon > 0$. Let $\bar{M} = \max\{M_1, M_2\}$. Let $||(b, \rho)||_S, ||(b, \rho)||_2$ denote the sup and euclidean norm. Since these are equivalent norms, there exists a constant $\kappa > 0$ such that $||(b, \rho)||_S \leq \kappa ||(b, \rho)||_2$. Let us consider $\delta = \epsilon / 2\kappa \bar{M}$. Then:

$$|\sigma_w^n(b_1, \rho_1) - \sigma_w^n(b_2, \rho_2)| = \left| \frac{\partial \sigma_w^n}{\partial b}(\tilde{b}, \tilde{\rho})(b_2 - b_1) + \frac{\partial \sigma_w^n}{\partial \rho}(\tilde{b}, \tilde{\rho})(\rho_2 - \rho_1) \right|$$

$$\leq M_1(b_2 - b_1) + M_2(\rho_2 - \rho_1) \leq \bar{M}||(b_1, \rho_1) - (b_2, \rho_2)||_S \leq \kappa \bar{M}||(b_1, \rho_1) - (b_2, \rho_2)||_2.$$

Hence, if $||(b_1, \rho_1) - (b_2, \rho_2)||_2 < \delta$ this implies that $|\sigma_w^n(b_1, \rho_1) - \sigma_w^n(b_2, \rho_2)| < \epsilon$. Since this occurs for every $n \in \mathbb{N}$, this is an equicontinuous sequence of functions. Then, by the Arzelà-Ascoli Theorem, this sequence must have a convergent sub-sequence. Since this happens for an arbitrary sequence in Σ_w, this set is compact.

In order to show that this game has an equilibrium, I need to show that the mapping $\tilde{\pi} : \Sigma_w \to \Sigma_w$ given by:

$$\tilde{\pi}(\sigma_w)(b, \rho) = \rho \pi^{\xi^P}(b, \rho | \sigma_w) + (1 - \rho) \pi^{\xi^I}(b, \rho | \sigma_w)$$

has a fixed point. To do this, I once again will use the Schauder Fixed Point Theorem. To use the theorem, I will show that, individually, each mapping $\pi^{\xi}(\cdot | \sigma) \in \Sigma_w$ and that it is continuous.

For this proof, I will make the simplification of linearizing the real interest rate, so instead of considering $(1 + r)(1 + \pi^e)b/1 + \pi$, I consider $1 + r + \pi^e - \pi$ to be the real interest rate. This has the advantage of simplifying the first order conditions, and since$\pi \in [0, 1]$ it is a good approximation of $(1 + r)(1 + \pi^e)b/1 + \pi$. This assumption also allows me to get rid of expectations for the imprudent government's problem (since the FOCs are linear in ϵ_π, ϵ_d and hence these are redundant).

I begin with the problem of the imprudent government. An advantage of this problem is that, since ξ^I is myopic, only σ_w affects ξ^I's problem, i.e., the decisions of the prudent government are not relevant for the imprudent government (this is not true for the prudent government's problem). The imprudent government's problem is:

$$\max_{\pi, d} \; -((1 - k)\bar{y} - \theta(\pi - \sigma_w(b, \rho)) + d)^2 - s(\pi - \bar{\pi})^2 - \gamma(d + (1 + r + \sigma_w(b, \rho) - \pi)b - \bar{m}\pi)^2.$$

The first order conditions of this problem are:

$$\theta\left((1 - k)\bar{y} + \theta(\pi - \sigma_w(b, \rho) + d) + s(\pi - \bar{\pi}) = \gamma(b + \bar{m})\left(d + (1 + r + \sigma_w(b, \rho) - \pi)b - \bar{m}\pi\right),\right.$$

$$((1 - k)\bar{y} + \theta(\pi - \sigma_w(b, \rho) + d) = -\gamma\left(d + (1 + r + \sigma_w(b, \rho) - \pi)b - \bar{m}\pi\right).$$

Let $A = 8(1 + \gamma)(\theta^2 + s + \gamma(b + \bar{m})^2) + 4(\theta + \gamma(b + \bar{m}))^2 > 0$. Then, using the implicit function theorem:

$$\frac{\partial \pi^{\xi^I}}{\partial b}(b,\rho) = \frac{4}{A}\left[(1+\gamma)\left(\theta\frac{\partial \sigma_w}{\partial b}(b,\rho) - \gamma b' - (b+\bar{m})\right) + (\theta + \gamma(b+\bar{m}))(\theta+\gamma)\frac{\partial \sigma_w}{\partial b}(b,\rho)\right]$$

Since $b' \geq 0$ and $b + \bar{m} > 0$ then:

$$\frac{\partial \pi^{\xi^I}}{\partial b}(b,\rho) \leq \frac{B}{A}\frac{\partial \sigma_w}{\partial b}(b,\rho),$$

with $B = 4\theta(1+\gamma) + 4(\theta+\gamma(b+\bar{m}))(\theta+\gamma)$. By expanding the terms of A and B, one can show that $B < A$, and hence the derivative of π^{ξ^I} with respect to b is uniformly bounded by M_1. Now, again by the implicit function theorem:

$$\frac{\partial \pi^{\xi^I}}{\partial \rho}(b,\rho) = \frac{4}{A}\left[(1+\gamma)\left(\theta\frac{\partial \sigma_w}{\partial \rho}(b,\rho) - \gamma b'\right) + (\theta + \gamma(b+\bar{m}))(\theta+\gamma)\frac{\partial \sigma_w}{\partial b}(b,\rho)\right]$$

which again implies that the derivative of π^{ξ^I} with respect to ρ is uniformly bounded by M_2.

Now, lets consider the problem of the prudent government:

$$V(b,\rho) = \max_{\pi,d} \mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[(1-\delta)f(\pi,d,b,\rho,\epsilon_\pi,\epsilon_d) + \delta V\left(g(\pi,d,b,\rho,\epsilon_\pi,\epsilon_d), h(\pi,d,b,\rho,\epsilon_\pi,\epsilon_d)\right)\right].$$

The first order conditions of this problem are (consider that the derivatives of f are linear in ϵ_π, ϵ_d and hence expectations can be dropped):

$$(1-\delta)f_\pi + \delta\mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[V_b g_\pi + V_\rho h_\pi\right] = 0,$$

$$(1-\delta)f_d + \delta\mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[V_b g_d + V_\rho h_d\right] = 0.$$

Using the implicit function theorem, we get that:

$$\frac{\partial \pi^{\xi^P}}{\partial b} = -\frac{(1-\delta)f_{\pi b} + \delta\mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[V_{bb}g_\pi + V_b g_{\pi b} + V_{b\rho}h_\pi + V_\rho h_{\pi b}\right]}{(1-\delta)f_{\pi\pi} + \delta\mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[V_b g_{\pi\pi} + V_\rho h_{\pi\pi}\right]},$$

$$\frac{\partial \pi^{\xi^P}}{\partial \rho} = -\frac{(1-\delta)f_{\pi \rho} + \delta\mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[V_{b\rho}g_\pi + V_b g_{\pi \rho} + V_{\rho\rho}h_\pi + V_\rho h_{\pi \rho}\right]}{(1-\delta)f_{\pi\pi} + \delta\mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[V_b g_{\pi\pi} + V_\rho h_{\pi\pi}\right]}.$$

Following the same arguments as in the dynamic game, and using the fact that g is linear and hence its second derivatives are zero, then:

$$\frac{\partial \pi^{\xi^P}}{\partial b} \leq -\frac{f_{\pi b}}{f_{\pi\pi}} \leq \kappa_b \frac{\partial \sigma_w}{\partial b} \leq M_1,$$

$$\frac{\partial \pi^{\xi^P}}{\partial \rho} \leq -\frac{f_{\pi \rho}}{f_{\pi\pi}} \leq \kappa_\rho \frac{\partial \sigma_w}{\partial \rho} \leq M_2,$$

where $\kappa_b, \kappa_\rho < 1$ are constants that depend on the model's parameters. Hence, the prudent government's inflation choices are continuous and uniformly bounded the same bounds as σ_w. In conclusion, $\pi(\cdot|\sigma_w)$ lives in the same compact function set as σ_w, and then I can invoke again the Schauder fixed point theorem to conclude that an equilibrium exists.

C.2 Proof of Proposition 6

First, I prove the following lemma.

Lemma 4. *Let $\rho'(b, \rho, \epsilon_\pi, \epsilon_d)$ be the reputation updating rule. Then this is strictly increasing in both (b, ρ) for a fixed $(\epsilon_\pi, \epsilon_d)$.*

Proof. Using the definition of the reputation update, and defining $\Delta\pi(b, \rho) = \pi^{\xi^P}(b, \rho) - \pi^{\xi^I}(b, \rho)$, $\Delta d(b, \rho) = d^{\xi^P}(b, \rho) - d^{\xi^I}(b, \rho)$, I can rewrite it as:

$$\rho'(b, \rho, \epsilon_\pi, \epsilon_d) = \frac{\rho g_\pi(\epsilon_\pi) g_d(\epsilon_d)}{\rho g_\pi(\epsilon_\pi) g_d(\epsilon_d) + (1-\rho) g_\pi(\Delta\pi(b, \rho) + \epsilon_\pi) g_d(\Delta d(b, \rho) + \epsilon_d)}$$

I consider the following properties of $\Delta\pi(b, \rho)$: it is increasing in b and decreasing in ρ. The same applies for the deficit difference between types. Then, for a fixed $(\epsilon_\pi, \epsilon_d)$, the updating rule is then:

$$\rho'(b, \rho, \epsilon_\pi, \epsilon_d) = \frac{A}{A + B(b, \rho)}.$$

Then, using the fact that both pdfs considered are normal:

$$\rho'_b(b, \rho) = \frac{AB(b, \rho) \left[\frac{\Delta\pi(b,\rho)(b,\rho)}{\sigma_\pi^2} \frac{\partial \Delta\pi(b,\rho)}{\partial b} + \frac{\Delta d(b,\rho)(b,\rho)}{\sigma_d^2} \frac{\partial \Delta d(b,\rho)}{\partial b} \right]}{(A + B(b, \rho))^2}$$

Since $A, B, \Delta\pi, \Delta d > 0$ together with the properties of the derivatives of $\Delta\pi, \Delta d$, we get that $\rho'(b, \rho, \epsilon_\pi, \epsilon_d)$ is increasing in b. Similarly for the derivative of ρ' with respect to ρ. □

With this I can now prove Proposition 6.

1. $V^\xi(\cdot, \cdot)$ is a strictly concave and differentiable function for all (b, ρ) and $\xi \in \{\xi^P, \xi^I\}$.

   Proof. Strict concavity, and differentiability of V follows from the corollaries of the Maximum Theorem discussed in Stokey et al. (1989). These results follow from the properties of the function f. □

2. For every $\rho \in [0, 1]$, $V^{\xi^P}(\cdot, \rho)$ is strictly decreasing in b.

   Proof. The envelope condition for ξ^P's problem states that $V_b^{\xi^P}$ is equal to the derivative of the Lagrangean evaluated at the optimal decisions. Let $\lambda_1 > 0$ be the Lagrange multiplier associated to the b' restriction, and $\lambda_2 > 0$ the multiplier associated to the ρ' restriction, and $\lambda_3 > 0$ be the multiplier for the output restriction on Equation (2). Then:

$$\frac{\partial V^{\xi^P}}{\partial b}(b, \rho) = \frac{\mathcal{L}}{\partial \rho} =$$

$$\mathbb{E}_{\epsilon_\pi, \epsilon_d} \left[-\lambda_3 \sigma_b(b, \rho) - \lambda_2 \rho'_b(b, \rho, \epsilon_\pi, \epsilon_d) - \lambda_2 \frac{(1+r)\sigma_b(b, \rho) b + (1+r)(1 + \sigma(b, \rho))}{1 + \tilde{\pi}} \right] < 0,$$

   since $\sigma(b, \rho)$ is increasing in b and $\rho'(b, \rho, \epsilon_\pi, \epsilon_d)$ is increasing in b. □

3. For every $b \in [0, \bar{b}]$, $V^{\xi^P}(b, \cdot)$ is strictly increasing in ρ while $V^{\xi^I}(b, \cdot)$ is strictly decreasing.

   Proof. The envelope condition for ξ^P's problem states that $V_\rho^{\xi^P}$ is equal to the derivative of the Lagrangean evaluated at the optimal decisions:

$$\frac{\partial V^{\xi^P}}{\partial \rho}(b, \rho) = \frac{\mathcal{L}}{\partial \rho} =$$

$$\mathbb{E}_{\epsilon_\pi, \epsilon_d}\left[-\lambda_3 \sigma_\rho(b, \rho) + \lambda_2 \rho'_\rho(b, \rho, \epsilon_\pi, \epsilon_d) - \lambda_2 \frac{(1+r)\sigma_\rho(b, \rho) b}{1+\tilde{\pi}}\right] > 0,$$

   since $\sigma(b, \rho)$ is decreasing in ρ and $\rho'(b, \rho, \epsilon_\pi, \epsilon_d)$ is increasing in ρ.

   □

C.3 Proof of Proposition 7

The flow-payoff function for type ξ^I is:

$$f^{\xi^I}(\tilde{\pi}, \tilde{d}, b, \rho, \epsilon_\pi, \epsilon_d) = -\left((1-k)\bar{y} + \theta(\tilde{\pi} - \sigma_w(b, \rho) + \tilde{d}\right)^2 - s_I(\tilde{\pi} - \bar{\pi})^2 - \gamma\left(\tilde{d} + \frac{(1+r)(1+\sigma_w(b, \rho) b}{1+\tilde{\pi}} - \bar{m}\tilde{\pi}\right),$$

where $\tilde{\pi} = \pi + \epsilon_\pi$, $\tilde{d} = d + \epsilon_d$. The flow-payoff for the prudent type is the same but replacing s_I for s_P. Then:

$$f^{\xi^P}(\tilde{\pi}, \tilde{d}, b, \rho, \epsilon_\pi, \epsilon_d) = f^{\xi^I}(\tilde{\pi}, \tilde{d}, b, \rho, \epsilon_\pi, \epsilon_d) - (s_P - s_I)(\pi - \bar{\pi})^2. \quad \text{(C8)}$$

Since type ξ^I is myopic, $(\pi^{\xi^I}(b, \rho), d^{\xi^I}(b, \rho))$ maximize the expected value of $f^{\xi^I}(\pi, d, b, \rho)$, satisfying the first order condition

$$\mathbb{E}_{\epsilon_\pi, \epsilon d}\left[\nabla f^{\xi^I}\left((\pi^{\xi^I}(b, \rho), d^{\xi^I}(b, \rho), b, \rho, \epsilon_\pi, \epsilon_d\right)\right] = 0.$$

Let us assume, for the sake of contradiction, that there exists (b, ρ) such that $(\pi^{\xi^P}(b, \rho), d^{\xi^P}(b, \rho)) = (\pi^{\xi^I}(b, \rho), d^{\xi^I}(b, \rho))$. This implies that $\rho' = \rho$ and hence the first order conditions for type ξ^P are:

$$(1-\delta_P)\mathbb{E}_{\epsilon_\pi, \epsilon d}\left[\frac{\partial f^{\xi^P}}{\partial \pi}\right] + \delta_P \mathbb{E}_{\epsilon_\pi, \epsilon d}\left[\frac{\partial V^{\xi^P}}{\partial b'} \frac{\partial b'}{\partial \pi}\right] = 0,$$

$$(1-\delta_P)\mathbb{E}_{\epsilon_\pi, \epsilon d}\left[\frac{\partial f^{\xi^P}}{\partial d}\right] + \delta_P \mathbb{E}_{\epsilon_\pi, \epsilon d}\left[\frac{\partial V^{\xi^P}}{\partial b'} \frac{\partial b'}{\partial d}\right] = 0,$$

Using Equation (C8) notice that:

$$\mathbb{E}_{\epsilon_\pi, \epsilon d}\left[\frac{\partial f^{\xi^P}}{\partial d}\right] = \mathbb{E}_{\epsilon_\pi, \epsilon d}\left[\frac{\partial f^{\xi^I}}{\partial d}\right] = 0,$$

this last part due to $(\pi^{\xi^I}(b, \rho), d^{\xi^I}(b, \rho))$ being optimal for ξ^I and type ξ^P pooling. On the other hand, $\partial b'/\partial d > 0$ and $\partial V^{\xi^P}/\partial b' < 0$, which then implies:

$$0 = (1-\delta_P)\mathbb{E}_{\epsilon_\pi,\epsilon d}\left[\frac{\partial f^{\xi^P}}{\partial d}\right] + \delta_P \mathbb{E}_{\epsilon_\pi,\epsilon d}\left[\frac{\partial V^{\xi^P}}{\partial b'}\frac{\partial b'}{\partial d}\right]$$

$$= \delta_P \mathbb{E}_{\epsilon_\pi,\epsilon d}\left[\frac{\partial V^{\xi^P}}{\partial b'}\frac{\partial b'}{\partial d}\right] < 0,$$

which is a contradiction. Hence, no pooling can exist for any $(b,\rho) \in [0,\bar{b}] \times [0,1]$.

D Numerical Implementation

D.1 Dynamic Game

I break the explanation on how I implement my model in the computer into two parts: the first one will be about the solution of the government's problem, taking as given σ_w, and the second one will be about the solution of the wage setters' problem to find the optimal $\sigma_w^\star$.

D.1.1 Government's Problem Given σ_w

The government solves the following problem to find its best-response to the strategy of wage setters:

$$V(b|\sigma_w) = \max_{(\pi,d,b)} \; (1-\delta)f(\pi,d,b|\sigma_w) + \delta V\left(d + \frac{(1+r)(1+\sigma_w(b,\rho))b}{1+\pi} - \bar{m}\pi \,\middle|\, \sigma_w\right). \quad \text{(D9)}$$

Since σ_w is fixed, the government's problem is a standard dynamic programming problem. However, given that to solve the wage setters' problem I will need to re-solve Equation (D9) for each variation in σ_w, I need to able to find the government's problem solution as efficiently as possible. In order to do this, I used the theoretical characterization of the equilibrium I provided in Proposition 3. Since V is a smooth function, I can use a projection method, as in Boyd (2001), in order to approximate V. In this case, I considered a base of polynomials which (by the Stone-Weierstrass theorem) can approximate any continuous function.[14] Given the smoothness of b, I do not need to make sure the Bellman equation holds for all possible $b \in \mathcal{D}$, so I considered a finite grid for values of b, $\mathcal{G}$. This reduces (considerably) the complexity of the problem. I consider the following Value Function Iteration algorithm:

1. Consider $V_0 = 0$.
2. For each $b \in \mathcal{G}$, define V_n^{aux} as:

   $$V_n^{aux}(b|\sigma_w) = \max_{(\pi,d)} \; (1-\delta)f(\pi,d,b|\sigma_w) + \delta V_{n-1}\left(d + \frac{(1+r)(1+\sigma_w(b,\rho))b}{1+\pi} - \bar{m}\pi \,\middle|\, \sigma_w\right).$$

   Also, keep track of the optimal inflation choices for the government $\pi_n^{aux}(b|\sigma_w)$ for each $b \in \mathcal{G}$.

14 Since $\mathcal{D}$ is a convex and compact subset of $\mathbb{R}$, the conditions of the Stone-Weierstrass theorem are satisfied.

3. Considering polynomials up until degree N_V, interpolate $\{(b, V_n^{aux}(b|\sigma_w))\}_{b\in\mathcal{G}}$ and let V_n be the function that interpolates this set. Similarly, define $\pi_n(b|\sigma_w)$ as the function that interpolates $\{(b, \pi_n^{aux}(b|\sigma_w))\}_{b\in\mathcal{G}}$.

4. Repeat steps 2 and 3 until $||V_n - V_{n-1}|| < \epsilon$.

D.1.2 Wage Setters' Problem

To solve the wage setters' problem, I considered two different methods, and then compared the solutions. For both I get the same solution as well as if I consider perturbations around it, they converge to the same method. This is what is suggesting to me that the model has a unique solution.

D.1.3 Fixed-Point Approach

This approach solves using the fact that the equilibrium is a fixed point of the best reply function. I know that any equilibrium strategy for wage setters must satisfy $\sigma_w(b) = \pi(b|\sigma_w)$ for all $b \in \mathcal{D}$. Hence, the equilibrium's wage setters strategy must be a solution to the following problem:

$$\max_{\sigma_w:\mathcal{D}\to\mathcal{D}} \int_{\mathcal{D}} -\left(\pi(b|\sigma_w) - \sigma_w(b)\right)^2 db,$$
$$\pi(b|\sigma_w) \in \Gamma(b|\sigma_w).$$

This is a variational problem, so in order to approximate its solution, I follow Ritz's method, described in Gelfand and Fomin (1963), which is guaranteed to work since $\sigma_w^\star$ is a smooth function. In a nutshell, Ritz's method consists in approximating the solution to a variational problem by a function that belongs to a finite-dimensional space. In this case, I considered a base of polynomials which can approximate any continuous function. This turns a variational problem into a "simple" optimization problem in which the unknowns are the coefficients of the polynomials. Let $\sigma_w^N(b) = a_0 + a_1 b + ... + a_N b^N$ be the approximation of $\sigma_w^\star$, and let $\pi\left(b|\sigma_w^N\right)$ be the government's best reply to σ_w^N. Then, I solve the following problem:

$$\max_{a_0,a_1,...,a_N} \int_{\mathcal{D}} \left(\pi\left(b|\sigma_w^N\right) - \sigma_w^N(b)\right)^2 db,$$

where $\pi\left(b|\sigma_w^N\right)$ is the policy function obtained from the algorithm described in Appendix D.1.1, taking $\sigma_w^N(b)$ as given.

D.1.4 Recursive Approach

As Theorem 1 shows, the equilibrium is a fixed point of the mapping $\Pi : \Sigma \to \Sigma$. If this mapping were to be a contraction, then the fixed point could be easily found by finding the limit of the sequence $\sigma_w^n = \Pi(\sigma_w^{n-1})$ for any $\sigma_w^0 \in \Sigma$. Unfortunately, I have no indication that these mapping is indeed a contraction. Doing this iterative procedure is not guaranteed to converge.

I consider the following recursive approach and then compare the solutions I get with the fixed-point approach:

1. Consider $\sigma_w^0(b) = b^2 \in \Sigma$.

2. Using the algorithm described in Appendix D.1.1, find π^n taking as given σ_w^{n-1}.

3. Define $\sigma_w^n = \pi^n$ and repeat until $||\sigma^n - \sigma^{n-1}|| < \epsilon$, where $||\cdot||$ is the supremum norm.

This algorithm leads to the same solution as in the fixed point approach, even if I consider different σ_w^0. This could be indicative of Π being a contraction, however, I have not yet found a way of showing this.

D.2 Reputation Game

The numerical implementation I follow to solve for this model, is similar to the one that I used to solve the dynamic game. The main difference is that now I need to solve two optimization problems for the government (one for each type). But, given that type ξ^I is myopic, its problem is trivial to solve. The main computational complexity challenge is in solving type ξ^P's problem for a given (σ_w, σ_G^I). The main difference in the method presented in Appendix D.1.1 is that now I consider a tensor product of polynomials to approximate the value function, since now the state space is two-dimensional. The rest of the algorithm is the same.

The additional challenge in the reputation game is that I now have a "fixed point problem within a fixed point problem" in the sense that the equilibrium $(\sigma_w, \pi^{\xi^P}, \pi^{\xi^I})$ have to be a best replies to each other, and the updating rule is itself a function of these functions.[15] So, for example, here is the modification of the recursive approach I use to solve this model:

1. Consider $\sigma_w^0 = \pi^{\xi^{P,0}} = \pi^{\xi^{I,0}} = (1-\rho)b^2$.

2. Taking as given $(\sigma_w^{n-1}, \pi^{\xi^{P,n-1}})$ let $\pi^{\xi^{I,n}}$ be the solution of ξ^I's problem.

3. Taking as given $(\sigma_w^{n-1}, \pi^{\xi^{I,n}})$ let $\pi^{\xi^{P,n}}$ be the solution of ξ^I's problem.

4. Taking as given $(\pi^{\xi^{P,n}}, \pi^{\xi^{I,n}})$ let σ_w^n be defined as:

$$\sigma_w^n(b,\rho) = \rho\pi^{\xi^{P,n}}(b,\rho) + (1-\rho)\pi^{\xi^{I,n}}(b,\rho).$$

5. Repeat this procedure until $||(\sigma_w^n, \pi^{\xi^{P,n}}, \pi^{\xi^{I,n}}) - (\sigma_w^{n-1}, \pi^{\xi^{P,n-1}}, \pi^{\xi^{I,n-1}})|| < \epsilon$.

Again, even after considering different initial functions, the algorithm converges to the same solution.

15 This is a common problem across all reputation models, see Mailath and Samuelson (2006).