Public Good Provision and Optimal Taxation in a Hidden Income World

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Motivation

- According to World Bank Data, in 2018 the average of tax recollection as a percentage of GDP in developing economies is 18% while in developed economies it is 30%.
- One explanation: developing economies have large informal sectors, in which enforcing tax recollection is highly costly.
  - According to the International Labour Organization, in 2018 the average rate of informal employment in developing economies was 48%.
- Another issue that comes with informality is the free-rider problem: tax recollection is used in government spending which has to consider the entire population of the economy, not just the ones who pay taxes.

Motivation

- Why does informality arise?
  - Informality is usually defined as a situation in which a worker is not enrolled in a social security network.[1]
  - In this case, the government has no way of directly observing the income of informal workers, which makes taxation difficult.
  - So, maybe one explanation of why informality exists is that joining the social security system is not that appealing in developing economies because it does not have good returns and involves paying taxes.
- What is the optimal tax system in the presence of hidden income/informality? Is zero informality optimal?

[1] This is the definition followed by the ILO, IMF, among others.

My Proposal

- In order to answer these questions I propose a hidden income model with the following ingredients:
  - The government can only observe the amount saved by agents in the financial system of the economy (namely, banks).
  - Agents live for two periods (young/old) and when young must decide how much to save for retirement, either in the bank or in the couch.
  - Taxes are collected from agents when young and they are used to construct a public good that will benefit consumers when old.
- The main trade-off the government faces when designing the tax system is that higher taxes push people out of the financial system, which in turn generates them a lower income when old, due to an underprovision of the public good.

Related Literature

- Mirrlees Taxation: Kocherlakota (2005, 2010), Farhi and Gabaix (2020), Heathcote and Tsujiyama (2021).
- Hidden Income: Cole and Kocherlakota (2001), Doepke and Townsend (2006), Basetto and Phelan (2008).
- Public Goods Provision with Frictions: Mailath and Postlewaite (1990), Posner and Weinstein (2007), Bierbrauer (2009).

Basic Ingredients

- We consider an economy with a continuum of agents, represented by the $[0, 1]$ interval.
- Agents live for two periods:
  - Young: period in which they receive an income, must pay taxes, and decide how much to save for the future.
  - Old: period in which agents consume using the saving they have accumulated as well as the provision of a public good.
- Each agent will be characterized by an exogenously given type θ , namely productivity, which is distributed according to the CDF $F(\cdot)$.
  - Agents are assigned their productivity at birth i.i.d. from other agents.
  - The productivity of each agent will determine her income when young, denoted $e(\theta)$, which we assume is a strictly increasing function of θ.

Basic Ingredients

- Taxes are recollected from each agent in the economy, in order to use them in the provision of a public good.
- The total amount of public good provided by the government, denoted g, will be the (weighted) average of agent's contributions.
- We are interesting in studying the optimal tax sequence in this environment considering two scenarios:
  1. The government can perfectly observe θ for every agent (and hence observe the agent's income).
  2. The government does not know the productivity of any agent, but can observe how much the agent is saving.
- In both cases, the government's objective will be to maximize welfare in the economy.

Full Information Model

- We begin by analyzing the scenario in which the government can perfectly observe the productivity of each agent.
- Hence, the government will be choosing a tax schedule $\tau : \Theta \to \mathbb{R}_+$ while the amount of public good provided is given by:

$$g = \int \tau(\theta)\ dF.$$

- Therefore, the utility of each agent will be given by:

$$u\left(e(\theta) - \tau(\theta) - b\right) + \beta u\left(b + \psi g\right),$$

where $b \geq 0$ represents the amount of savings the agent decides to make given τ and g, and $\psi > 1$ represents the positive externality that publics good have on the economy.

Full Information Model: Timing

- Timing in the model works as follows:
  1. Nature assigns each agent a productivity type according to F.
  2. The government **commits** to a tax schedule $\tau : \Theta \to \mathbb{R}_+$ seeking to maximize welfare in the economy.
  3. Upon observing τ, g each agent chooses how much to save, denoted $b(\theta)$, in order to maximize her utility.
- Note: in this framework each agent's contribution has a negligible impact on the amount of public good provided by the government. Hence, each agent can take as given g and solve for $b(\theta)$.

Full Information Model

Government's Problem

Hence, in this model the government solves the following maximization problem:

$$\max_{\tau:\Theta\to\mathbb{R}_+} \int u\left(e(\theta)-\tau(\theta)-b(\theta)\right)+\beta u\left(b(\theta)+\psi g\right)\ dF,$$

$$b(\theta)\in \underset{b\geq 0}{\operatorname{argmax}}\ \ u\left(e(\theta)-\tau(\theta)-b\right)+\beta u\left(b+\psi g\right)\ \ \forall\ \ \theta\in\Theta,$$

$$g=\int \tau(\theta)\ dF.$$

Proposition

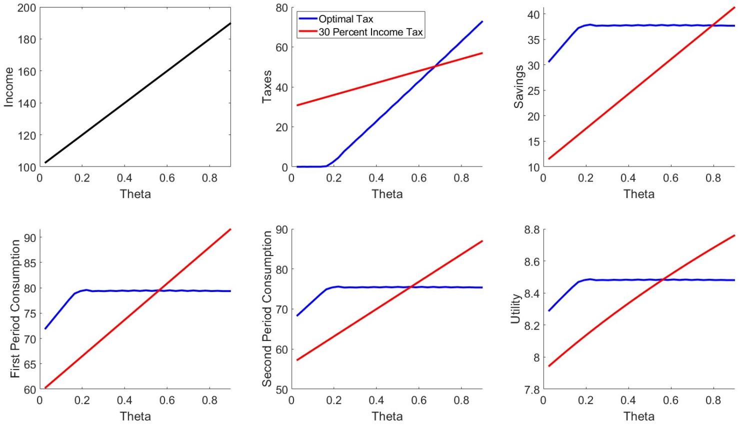
If $u(\cdot)$ is strictly concave then the government's problem has a solution, and for each τ, g pair there is a unique $b(\theta)$ that maximizes each agent's utility.

Full Information Model

Characteristics of the Optimal Solution

1. The optimal tax schedule $\tau^\star(\cdot)$ is weakly increasing in θ.
2. There is a cut-off $\bar{\theta}$ such that if $\theta \leq \bar{\theta}$ then $\tau^\star(\theta) = 0$.
3. Consumption when young and old is equal across all θ types larger than $\bar{\theta}$.

Full Information Model



Note: This graph considers $\beta = 0.95$, $\theta \sim U([0,1])$, and $\psi = 1.01$.

Hidden Income Model

- Now, we turn to study a model in the government can no longer directly observe agent's income.

- Instead, the government can only observe the amount of each agent's savings in the financial system of the economy (namely a bank).
  - Banks pay $R > 1$ when old per unit saved by young agents.
  - Hence the government now has to design a tax schedule τ that is a function of the observed savings.

- Agents have the option of not depositing their money in the bank, but to instead save it (in their couch) with a return of 1.
- Then, the utility of a type θ agent is given by:

$$u\left(e(\theta) - b - b^c - \tau(b)\right) + \beta u\left(Rb + b^c + \psi g\right),$$

where b is the amount of savings the agent has deposited in the bank, and b^c is the amount of couch savings.

Hidden Income Model

- In this context, we call an agent **informal** if $b = 0$ and $b^c \geq 0$, while an agent is an **evader** if $b > 0$ and $b^c > 0$.
- What changes with respect to the full information framework?
  - First, now the shape of the tax schedule becomes relevant for every agent, since now it depends on the amount the agent saves.
  - This means that now the government has to solve a **variational problem** in which the decision is to choose a tax function.
    - ★ We will restrict our analysis to $\tau \in \mathcal{D}$, the set of differentiable functions with domain in $[0, \infty)$.
  - Also, in this framework, couch savings represent an outside option for agents, so the government needs to consider that higher taxes may push people towards the couch (which we will call evasion).

Hidden Income Model

Government's Problem

Hence, in this model the government solves the following maximization problem:

$$\max_{\tau \in \mathcal{D}} \int u\left(e(\theta) - b(\theta,\tau,g) - b^c(\theta,\tau,g) - \tau(b(\theta,\tau,g))\right) + \beta u\left(Rb(\theta,\tau,g) + b^c(\theta,\tau,g) + \psi g\right)\, dF$$

Subject to the restriction that $b(\theta,\tau,g), b^c(\theta,\tau,g)$ must solve:

$$\max_{\substack{b \geq 0 \\ b^c \geq 0}} \quad u\left(e(\theta) - b - b^c - \tau(b)\right) + \beta u\left(Rb + b^c + \psi g\right),$$

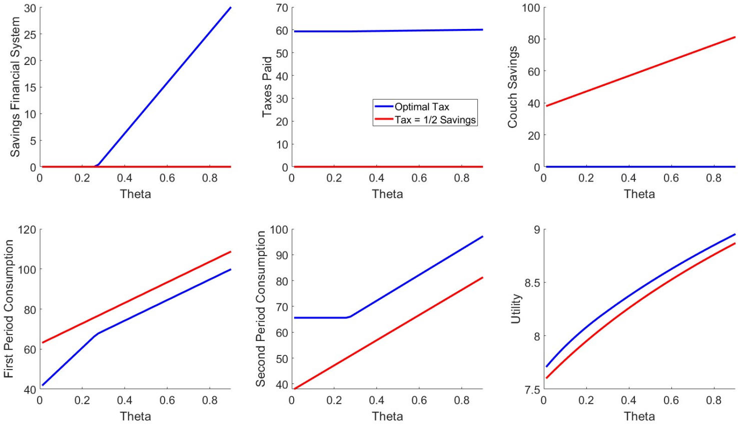
for every value of $\theta \in \Theta$, any tax function τ, and any value $g \geq 0$. In addition, there must be consistency:

$$g = \int \tau(b(\theta,\tau,g))\, dF.$$

Hidden Income Model: Next Steps

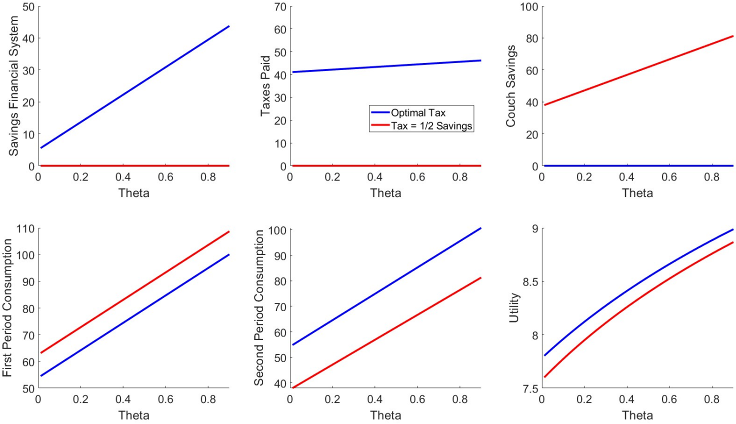
- In order to understand the model's dynamics I will take (first) some baby steps:
  - Consider the case in which τ can only be a linear function of b.
  - Start by solving the model in two extreme cases that simplify things: $R = 1$ and $R \to \infty$.
- These exercises had shed some light on how the model works and have highlighted to me some interesting results (which I hope hold in general):
  - Lower interest rates make the couch too attractive, so even in the optimal tax scheme there are agents who do not save in the financial system. With low interest rates, the optimal tax scheme is a lump sum tax.
  - As interest rates get higher, the informality is reduced but another interesting phenomenon arises at the top of the distribution: they begin to have incentives to not save everything in the financial system. Hence, the optimal tax system needs to incentivize them not to do this.

Hidden Income Model: Linear Taxes



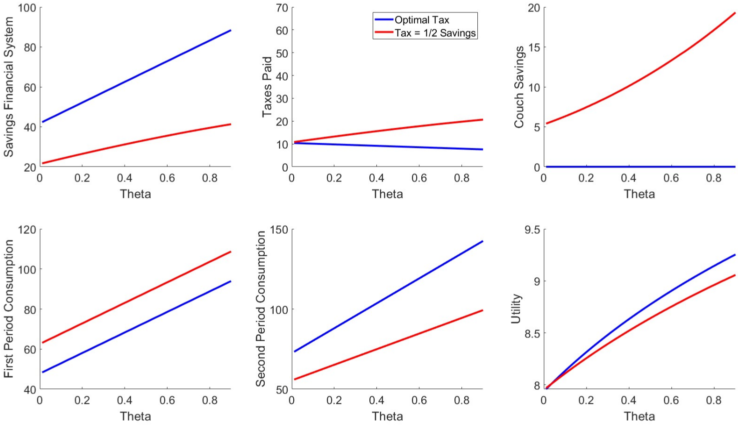
Note: This graph considers $\beta = 0.95$, $\theta \sim U([0,1])$, and $R = 1.05$.

Hidden Income Model: Linear Taxes



Note: This graph considers $\beta = 0.95$, $\theta \sim U([0,1])$, and $R = 1.1$.

Hidden Income Model: Linear Taxes



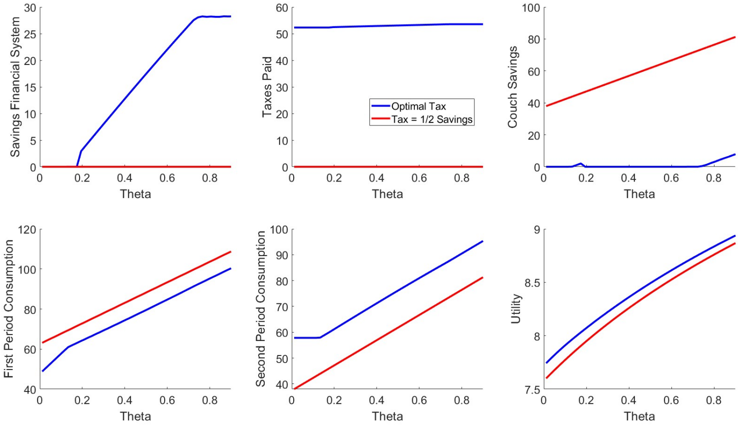
Note: This graph considers $\beta = 0.95$, $\theta \sim U([0,1])$, and $R = 1.2$.

Hidden Income Model: More General Case

- Now I present some preliminary results in which I consider a more general tax scheme.
  - I restrict $\tau(b)$ to be a polynomial of degree n, which allows me to turn the variational problem into a simpler problem of finding the coefficients of the polynomial.
  - Up until now I have solved (numerically) the model considering polynomials up to degree five, and have gotten similar results.

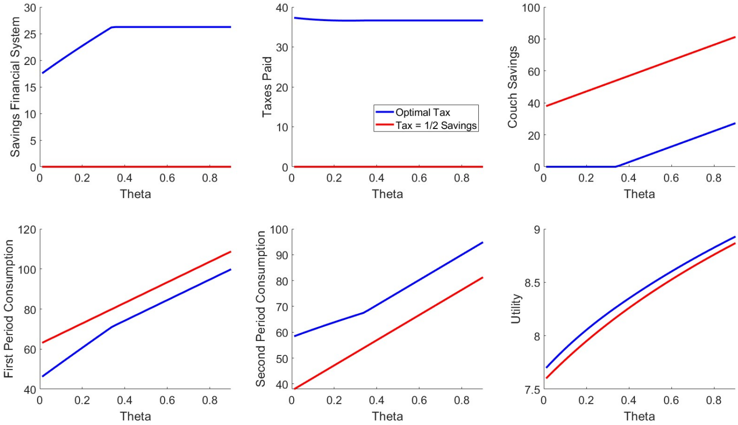
- The most interesting things to highlight:
  1. For low values of R the optimal tax schedule does not imply zero informality, and taxes are increasing and concave in b.
  2. Evasion in the upper part of the income distribution appears, even with low interest rates.
  3. Higher interest rates reduce informality but increase evasion in the upper part of the income distribution.

Hidden Income Model: More General Case



Note: This graph considers $\beta = 0.95$, $\theta \sim U([0,1])$, and $R = 1.05$.

Hidden Income Model: More General Case



Note: This graph considers $\beta = 0.95$, $\theta \sim U([0,1])$, and $R = 1.2$.

Future Work

- Here are my plans for the (immediate) future:
  1. Continue to explore the model's dynamics for a more general tax schedule.
  2. Use calculus of variations techniques to characterize the solution in the general model.
  3. Search for more data that can support my story and findings.
- Other possible extensions:
  - Both informality discussions as well as Mirrlees taxation focus usually on labor, so extending the model to incorporate labor decisions may be interesting.
  - Up until now the economy's interest rate is exogenous, which may not be a very good assumption given that savings are endogenous. So maybe it would be interesting to endogenize it.