Debt, Inflation, And Government Reputation

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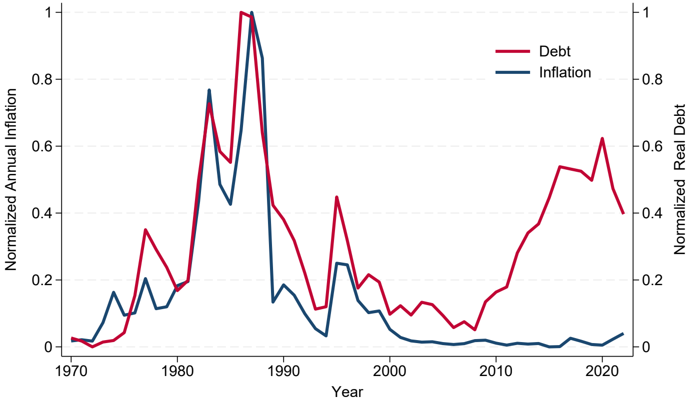
Motivation LATAM

- During the 20th century, many emerging economies experienced high debt levels that, in some episodes, were followed by high inflation; while in others, inflation was less responsive.



Motivation

- For a broader set of emerging economies, the correlation between inflation and debt declined after the establishment of an inflation targeting regime.

	Before IT	After IT	Last 10 Years
Corr. Debt and Inflation	0.83	0.13	0.36

- Agents' perception on the government's commitment to low inflation is key.

- I propose an incomplete information game, in which government reputation determines inflation-debt dynamics.

- **Government Reputation: probability agents assign to be facing a government that is committed to low inflation.**

My Proposal

- Game between private agents and a **consolidated government**, which makes decisions on both inflation and debt.
- I add fiscal considerations to Barro and Gordon (1983)'s monetary game.
- Main tension: a within-period time inconsistency, in which the government would like to surprise wage setters with higher inflation, to stimulate output and reduce debt.
  - This "incentive to surprise" is influenced by the state the economy is in.

Main Findings

- Inflation is highly correlated with public debt when government reputation is low, and less correlated when reputation is high.
- "Low Reputation Effect": even a government committed to low inflation may choose elevated inflation and deficits when it has low reputation.
- All this behavior is captured within a single separating equilibrium with no changing types.

Related Literature

- I propose a theory for inflation dynamics in which the time inconsistency problem is not solved but rather mitigated/worsened by both the joint evolution of debt and government reputation.
- Positive Theories of Inflation: Barro and Gordon (1983), Canzoneri (1985), and Bassetto and Miller (2023).
- Fiscal and Monetary Policy Interaction: Diaz Gimenez et. al. (2008), Sargent et. al. (2009), Kocherlakota (2012), Bianchi and Melosi (2017).
- Reputation in Policy Games: Backus and Driffill (1985), Dovis and Kirpalani (2021), Amador and Phelan (2021), Fourakis (2023).

Model: Stage Game

- Two players: wage setters, and a government.
- Time is discrete $t = 1, 2, 3, \ldots$
- In each period, both players choose their actions simultaneously:
  1. Wage setters choose w_t.
  2. The government chooses inflation (π_t) and deficit (d_t).
- The choice of these variables determines the price level p_t, output y_t, and the evolution of debt in the economy b_t.

Model: Stage Game

- Continuum of monopolistically competitive wage setters, where $i \in [0,1]$ chooses w_t^i to maximize:

$$UW_t^i = -\left(\frac{w_t^i - p_t}{p_{t-1}}\right)^2 = -(\pi_t - \pi_t^{e,i})^2.$$

Let $w_t = \int w_t^i di$ and $\pi_t^e = \int \pi_t^{e,i} di$. WS Problem

- Government chooses π_t, d_t: G Problem

$$UG_t = -(y_t - k\bar{y})^2 - s(\pi_t - \bar{\pi})^2 - \gamma b_t^2,$$

where:

$$y_t = \bar{y} + \theta(p_t - w_t) + d_t = \bar{y} + \theta(\pi_t - \pi_t^e) + d_t,$$

$$b_t = d_t + \frac{(1+r)(1+\pi_t^e)b_{t-1}}{1+\pi_t} - \bar{m}\pi_t.$$

Model: Dynamic Game

- Complete information framework: no reputational concerns.
- Dynamic game, since debt is a state variable.
- Since there is a continuum of wage setters and the government only observes aggregate behavior, they choose their wage myopically.
- The government chooses inflation and debt taking into account future implications of its decisions.
  - The government discounts the future with factor $\delta \in [0, 1)$.
- Perfect monitoring structure: at each t, players observe the history of the game up to that point.

Dynamic Game: Equilibrium Definition Existence

Definition Non-Markovian Equilibria

A Markov perfect equilibrium of this game are functions (π^e, π, d) such that:

1. (π^e, π, d) are Markov strategies.
2. Given (π, d), wage setters choose π^e to maximize their payoffs. WS Problem
3. Given π^e, the government chooses (π, d) to maximize its payoffs. G Problem

Equilibrium Characterization

Qualitative Features

Proposition

In a Markov Perfect Equilibrium of this dynamic game:

1. $\pi^{\star}(\cdot)$ is an increasing function of b.
2. $d^{\star}(\cdot)$ is a decreasing function of b.
3. No surprise inflation: $\pi^{e\star}(b) = \pi^{\star}(b)$ for all b, which implies:

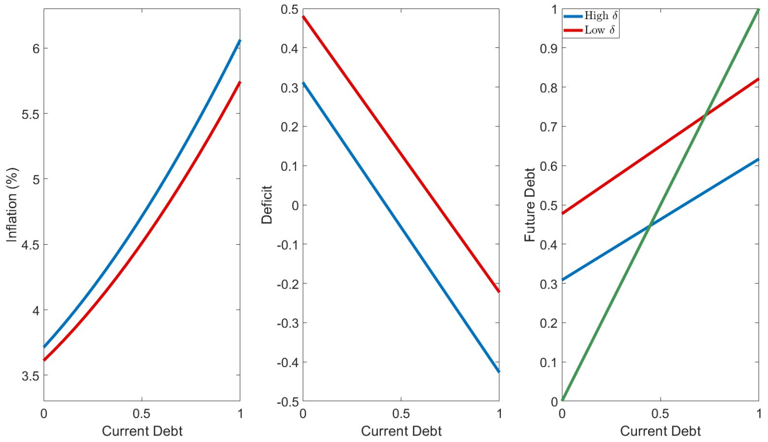
$$y = \bar{y} + \theta\left(\pi^{\star}(b) - \pi^{e}(b)\right) + d^{\star}(b) = \bar{y} + d^{\star}(b),$$

$$b' = d^{\star}(b) + \frac{(1+r)(1+\pi^{e}(b))b}{1+\pi^{\star}(b)} - \bar{m}\pi^{\star}(b) = d^{\star}(b) + (1+r)b - \bar{m}\pi^{\star}(b)$$

4. $V^{\star}(\cdot)$ is a decreasing function of b.

Equilibrium Example: The Role of δ Details

$$V(b) = \max_{\pi, d, b'} \; (1-\delta)\left[-(y - k\bar{y})^2 - s(\pi - \bar{\pi})^2 - \gamma(b')^2\right] + \delta V(b').$$



Equilibrium Example: The Role of s Details

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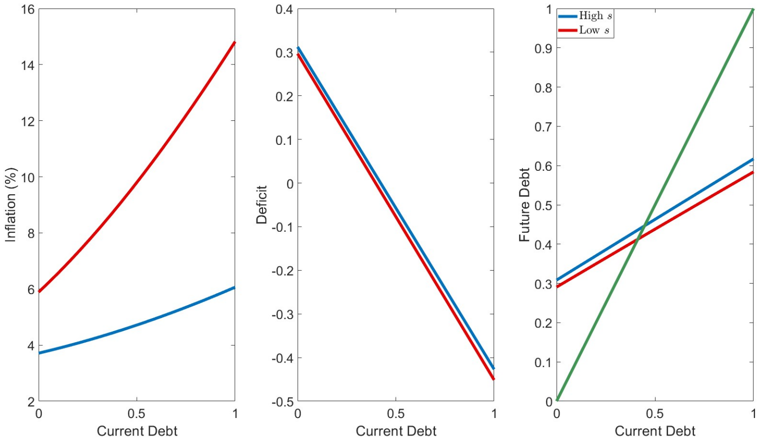
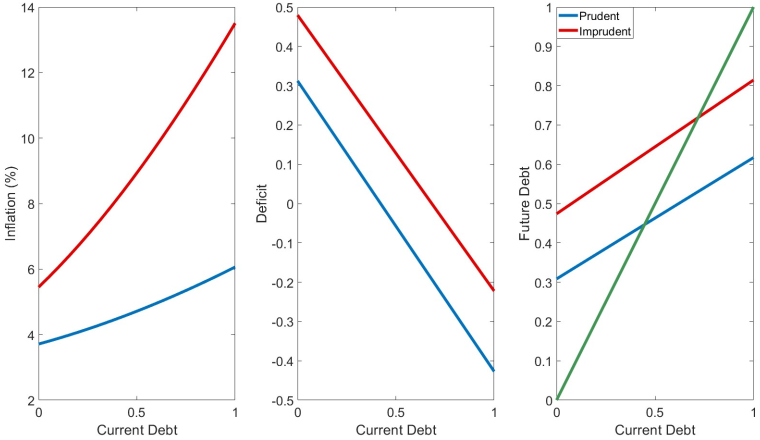


Example: Prudent vs Imprudent Governments Details

$$V(b) = \max_{\pi, d, b'} \; (1-\delta)\left[-(y-k\bar{y})^2 - s(\pi-\bar{\pi})^2 - \gamma(b')^2\right] + \delta V(b').$$



Takeaways of Dynamic Game

- A government with higher δ generates a lower debt policy function and a lower long-run debt level.
- A government with higher s generates a lower inflation policy function and a lower long-run inflation level.
- This model can explain, by varying δ and s, how a government can generate high inflation with high debt levels, or low inflation with low debt levels.
- The model has problems in explaining why a government would generate low inflation with high debt levels.
- Solution: introduce reputational concerns.

Reputation Framework

- Wage setters now interact with a government that may be of two types:
  1. Type P (Prudent): a government that has a discount factor $\delta_P \in (0, 1)$ and a disutility for inflation parameter s_P.
  2. Type I (Imprudent): a government that has a discount factor $\delta_I = 0 < \delta_P$ and $s_I < s_P$.
- Let $\rho_0 \in [0, 1]$ be the prior probability that the government is of type P.

Reputation Framework: Imperfect Monitoring

- Players observe a noisy signal of the government's actions:

$$\tilde{\pi} = \pi^{\xi} + \epsilon_{\pi}, \quad \tilde{d} = d^{\xi} + \epsilon_{d}, \quad \tilde{b}' = \tilde{d} + \frac{(1+r)(1+\pi^{e})b}{1+\tilde{\pi}} - \bar{m}\tilde{\pi},$$

  where ϵ_x are i.i.d. random variables with mean zero and variance σ_x^2.

- These are monetary and fiscal policy shocks, whose value is not known to either players at the moment they decide their actions.

- Shocks are relevant to have non-trivial reputation dynamics in equilibrium.

Reputation Framework: Timing

- At each period t, the game proceeds as follows:
  1. Upon the previous history of play, wage setters form a belief on the type of government they are facing.
  2. Both wage setters and the government choose their actions simultaneously.
  3. The shocks $(\epsilon_\pi, \epsilon_d)$ are realized, and players observe $(\tilde{\pi}_t, \tilde{d}_t, \tilde{b}'_t)$.
  4. The history of play for the next period will be given by $h^{t+1} = (h^t, \tilde{\pi}_t, \tilde{d}_t, \tilde{b}'_t)$.

Reputation Framework: Markov Strategies

- Once again, I restrict attention to analyze pure Markov strategies.
- In this dynamic game, there are two state variables:
  1. The current debt level b.
  2. Government reputation ρ, i.e., the belief of wage setters that the government is of type P given the observed history of play up until that point.

Reputation Framework: Equilibrium Definition

Definition

A Markov perfect equilibrium of this game are functions $(\pi^e, \pi^P, d^P, \pi^I, d^I, \rho')$ such that:

1. Given (π^P, d^P, π^I, d^I), wage setters choose π^e to maximize their payoffs. WS Problem
2. Given $(\pi^e, \pi^I, d^I, \rho')$ type P chooses (π^P, d^P) to maximize its payoffs. P Problem
3. Given $(\pi^e, \pi^P, d^P, \rho')$ type I chooses (π^I, d^I) to maximize its payoffs. I Problem
4. The updating rule ρ' is consistent with Bayes' Rule. Updating Rule

Reputation Framework: Equilibrium

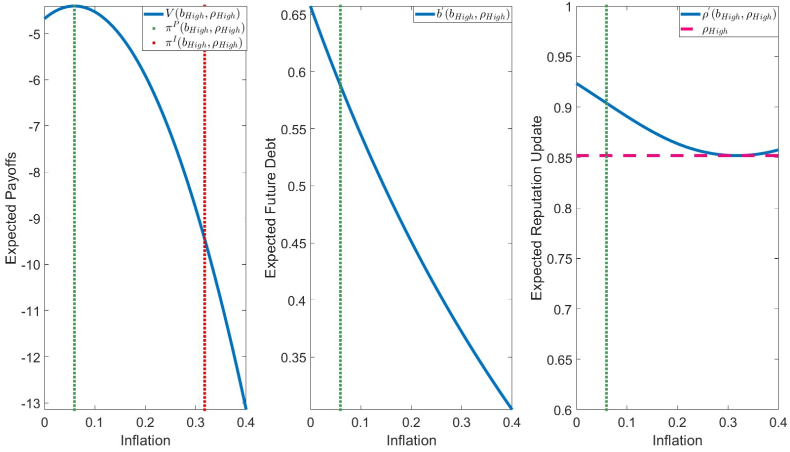
Theorem Proof

A perfect Markov equilibrium of this repeated game exists. Furthermore:

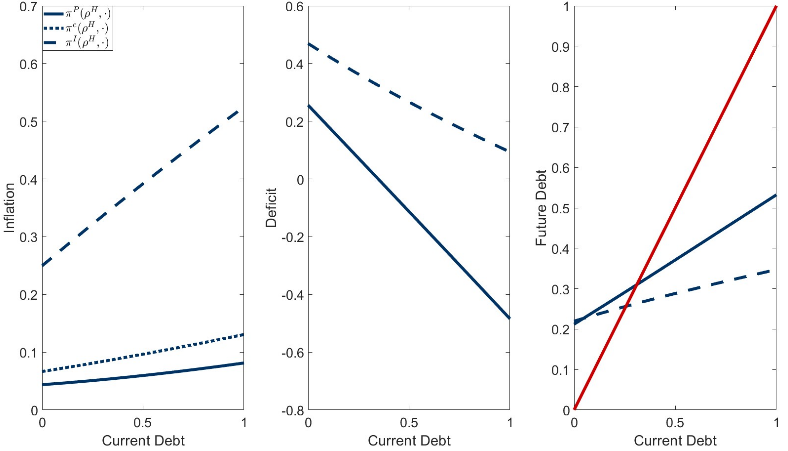
1. Any equilibrium requires separation.
2. In equilibrium:
$$\pi^e(b,\rho) = \rho\pi^P(b,\rho) + (1-\rho)\pi^I(b,\rho).$$

Reputation Framework: Equilibrium

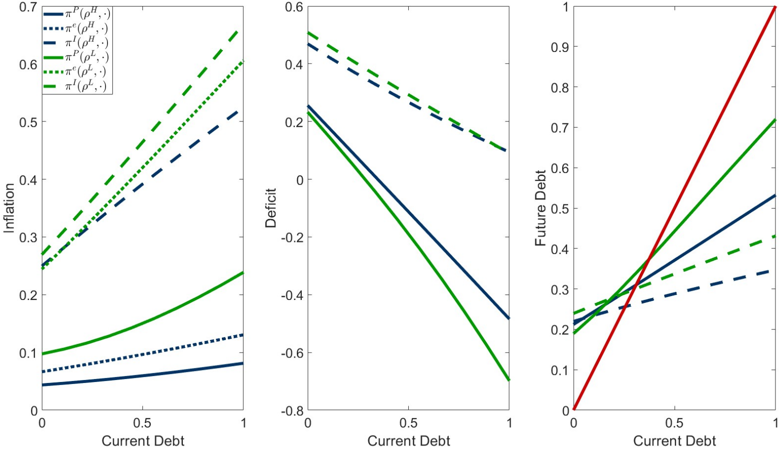
- The prudent government faces a trade-off of either increasing reputation or decreasing future debt.



Equilibrium: High Reputation



Equilibrium: High vs Low Reputation



Equilibrium: High vs Low Reputation

Low Reputation

- Why does a prudent government choose high inflation and deficit levels when it has low reputation?
- In equilibrium, whenever ρ is low, inflation expectations are close to $\pi^I(b,\rho)$.
- If the prudent government were to choose an inflation level that is considerably lower than $\pi^I(b,\rho)$, it would generate both a lower output and a higher debt:

$$y = \bar{y} + \theta\left(\pi^P(b,\rho) - \pi^e(b,\rho)\right) + d^P,$$

$$b' = d^P(b,\rho) + \frac{(1+r)(1+\pi^e(b,\rho))b_t}{1+\pi^P(b,\rho)} - \bar{m}\pi^P(b,\rho).$$

Qualitative Features of Equilibrium

Proposition

The following properties hold in a Markov perfect equilibrium:

1. Wage setters' inflation expectations:

$$\frac{\partial \pi^e}{\partial b}(b,\rho) > 0, \quad \frac{\partial \pi^e}{\partial \rho}(b,\rho) < 0, \quad \frac{\partial^2 \pi^e}{\partial b \partial \rho} < 0.$$

2. Prudent government:

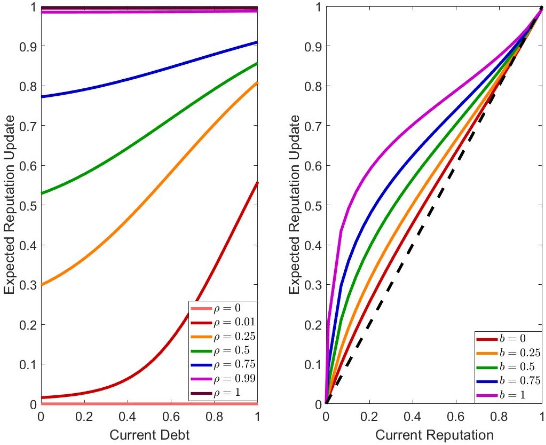
$$\frac{\partial \pi^P}{\partial b}(b,\rho) > 0, \quad \frac{\partial \pi^P}{\partial \rho}(b,\rho) < 0,$$

$$\frac{\partial V^P}{\partial b}(b,\rho) < 0, \quad \frac{\partial V^P}{\partial \rho}(b,\rho) > 0.$$

3. Imprudent government similar to prudent, except that I's payoffs are decreasing in ρ.

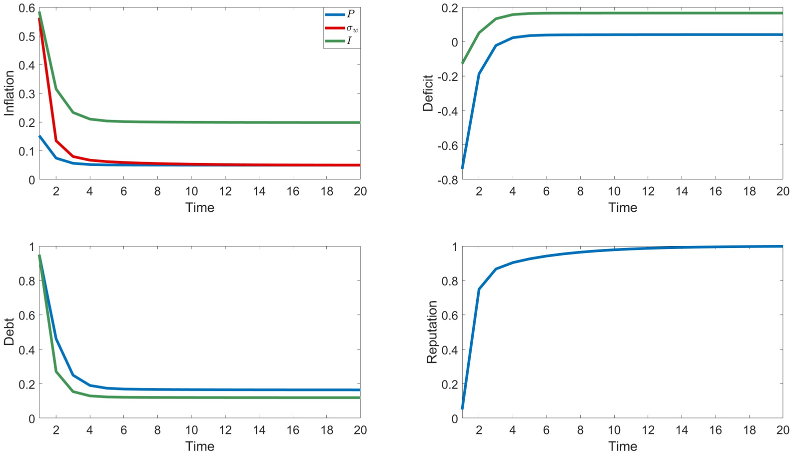
Updating Rule Dynamics
3D Updating Rule

- The prudent government is more likely to earn reputation in "bad times".



Equilibrium: Long-Run Learning

Learning with Noise



Model Predictions and Mexican Data

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- I calibrated the model choosing the parameter values that minimize the distance between the model's inflation expectations time series and the observed inflation time series. Parameter Values
- Then, I compare the model's predictions for deficit, output and expectations with the available data.

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$$d_t = b_t^{data} + \bar{m}\pi_t^{data} - \frac{(1+r)\left(1+\pi_t^e(b_t^{data}, \rho_t)\right) b_{t-1}^{data}}{1+\pi_t^{data}}.$$

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- For the reputation update, I considered the likelihood of receiving shocks of size $\pi_t^{data} - \pi_t^P(b_t^{data}, \rho_t)$, $d_t - d_t^P(b_t^{data}, \rho_t)$

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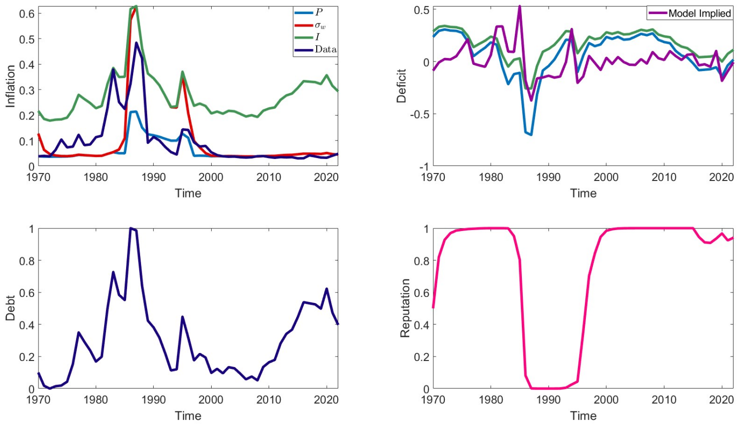
- For the fiscal deficit "data", I compute the fiscal deficit that is consistent with the model:

$$d_t = b_t^{data} + \bar{m}\pi_t^{data} - \frac{(1+r)\left(1+\pi_t^e(b_t^{data}, \rho_t)\right) b_{t-1}^{data}}{1+\pi_t^{data}}.$$

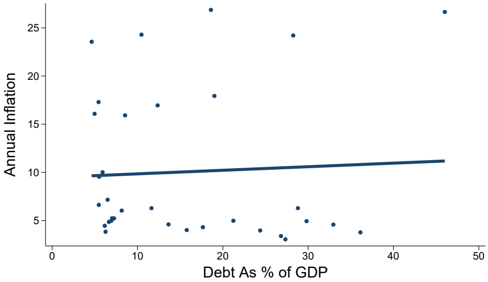
- For the reputation update, I considered the likelihood of receiving shocks of size $\pi_t^{data} - \pi_t^P(b_t^{data}, \rho_t)$, $d_t - d_t^P(b_t^{data}, \rho_t)$ vs $\pi_t^{data} - \pi_t^I(b_t^{data}, \rho_t)$, $d_t - d_t^I(b_t^{data}, \rho_t)$.

Model Predictions and Mexican Data

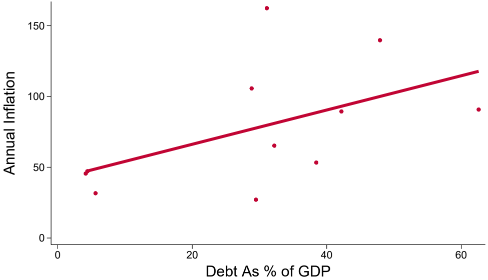
Details



Debt, Inflation, And Government Reputation in Mexico



High Reputation



Low Reputation

In Sample Prediction's Validation

- I considered data on inflation and debt to calibrate the model.
- I now compare the model's predictions with the (available) data on output gap (1970-2022), fiscal deficit (1990-2022), and inflation expectations (2000-2022).

Variable	M. Data/M. Model	Corr. Data and Model
Output Gap	1.23	0.47
Fiscal Deficit	0.94	0.89
Inflation Expectations	1.57	0.87

Key Takeaways of Reputation Model

- The value of reputation is crucial to determine government behavior.
  - Even a prudent government finds it optimal to choose high inflation and deficits when it has low reputation.
  - As its reputation increases, we should expect a disconnection between inflation and debt.
- The model predicts that inflation expectations capture the government's reputation.
  - If we observe an increase in debt and in expectations, this could be a signal of a deterioration in the government's reputation.

Debt, Inflation, And Government Reputation

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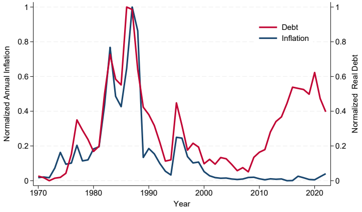
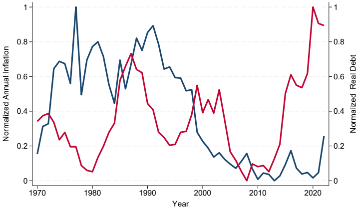
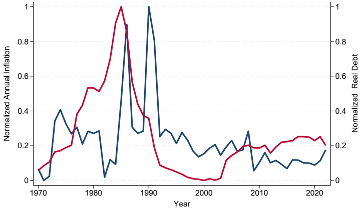
Debt and Inflation in Latin America [Back]



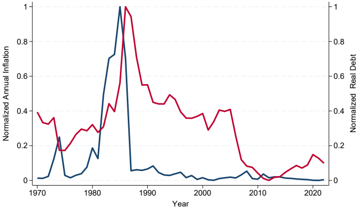
(a) Mexico.



(b) Colombia.



(c) Guatemala.



(d) Bolivia.

Model: Wage Setters

- I model the labor market as a monopolistic competition market, in which there is a continuum of wage setters, indexed by $i \in [0, 1]$.
- Wage setter i chooses w_t^i having as an objective a constant real wage over time:

$$UW_t^i = -\left(\frac{w_t^i - p_t}{p_{t-1}}\right)^2.$$

- Each i knows their wage decision does not affect p_t, and hence their expected utility maximizing wage choice is i´s expected price level, $w_t^i = p_t^{e,i}$.
- Since, by assumption, every wage setter has the same information when deciding w_t, then $w_t = p_t^{e,i} = p_t^e$ for all i.

Model: Wage Setters Back

- If we define inflation as $\pi_t = (p_t - p_{t-1})/p_{t-1}$, and $\pi_t^{e,i} = (p_t^{e,i} - p_{t-1})/p_{t-1}$ then we can re-write the wage setters payoffs as:

$$UW_t^i = -\left(\frac{w_t^i - p_t}{p_{t-1}}\right)^2 = -\left(\frac{p_t^{e,i} - p_{t-1} + p_{t-1} - p_t}{p_{t-1}}\right)^2 = -(\pi_t^{e,i} - \pi_t)^2.$$

- From now on, I consider $\pi_t^{e,i}$ to be the relevant variable for wage setters, which is pin downed by their wage decision.

Model: Government Back

- The government makes decisions on both the fiscal and monetary aspects of the economy.
- Each period, the government inherits a debt level b_{t-1}, and must decide on the deficit level d_t (government expenditures minus income) and growth rate of money g_t.
- The government is interested in pegging output, inflation, and debt to a target level:

$$UG_t = -(y_t - k\bar{y})^2 - s(\pi_t - \bar{\pi})^2 - \gamma b_t^2,$$

where $k > 1, s > 0, \gamma > 0$, $\bar{y}$ is the natural output level, and $\bar{\pi}$ is the inflation target of the government.

Model: Output Back

- Following the Neo-Keynesian tradition, output varies around a natural level $\bar{y}$.
- These fluctuations are driven by the labor market.
  - In an economy with sticky wages, higher prices attract more firms and workers to the market, increasing output.
- Hence, output is given by:

$$y_t = \bar{y} + \theta(p_t - w_t) + d_t = \bar{y} + \theta(\pi_t - \pi_t^e) + d_t,$$

where $\theta > 0$ measures the effect of the labor market on output.

Model: Evolution of Debt Back

- The evolution of government debt will be determined by:
  1. Real (primary) deficit d_t: the difference between government expenditures and income.
  2. Real service of debt: the amount of previous debt that the government must pay.
  3. Seigniorage: the revenue that the government gets from printing money.

- Importantly, according to the Fisher equation, the interest rate that the government faces is:

$$1 + i_t = (1 + r)(1 + \pi_t^e),$$

where $r > -1$ is the natural interest rate.

Model: Evolution of Debt Back

- The evolution of debt in real terms is given by:

$$b_t = d_t + \frac{(1+i_t)b_{t-1}}{1+\pi_t} - S_t = d_t + \frac{(1+r)(1+\pi_t^e)b_{t-1}}{1+\pi_t} - S_t.$$

- Then, higher inflation (as a result of higher money growth) generates a lower real interest rate and hence a higher seigniorage, which in turn will reduces the real value of debt.

Model: Restrictions on Debt Back

- I restrict debt to be non-negative, i.e., $b_t \geq 0$.
- Also, I assume that the government has a bound on the amount of debt it can issue $\bar{b}$.
- I focus on the case $\bar{b}$ is large and not-binding, although future work where $\bar{b}$ is binding could be interesting.
- Let $\mathcal{D} = [0, \bar{b}]$ be the set of feasible debt levels.

Model: Real Balances Demand and Money Growth Back

- To close up the model, I need to specify how prices are determined.
- Again, following the Neo-Keynesian tradition, I assume that prices and money are related through a money demand function.
- To keep the model simple, I assume that the government faces the following money demand function:

$$\frac{m_t}{p_t} = \bar{m},$$

which has two implications:

1. $g_t - \pi_t = 0$, i.e., by choosing g_t the government is pinning down inflation.
2. Seigniorage is then given by $S_t = \bar{m}\pi_t$.

Model: Markov Strategies Back

- I make a restriction to stationary Markov pure strategies.
  - Wage setters choose a strategy $\pi^e : \mathbb{R} \to \mathbb{R}$, where $\pi^e(b)$ represents inflation expectations upon observing a previous debt level b.
  - The government chooses $\sigma_g : \mathbb{R} \to \mathbb{R}^3$, where

    $$(\pi(b), d(b), b'(b))$$

    represent the inflation, deficit, and debt decisions upon observing b and given the strategy σ_w of wage setters.
- I restrict the strategy of wage setters to be a concave and twice differentiable function with uniformly bounded first derivatives.

Dynamic Game: Wage Setters' Problem Back

- Given the current debt b and a conjecture on government inflation behavior $\hat{\pi}$, the wage setters best reply is such that:

$$\pi^e(b) = \underset{\pi^e}{\text{argmax}} \quad -(\pi^e - \hat{\pi}(b))^2.$$

Dynamic Game: Government's Problem Back

- Given the current debt b and a conjecture on wage setters' behavior $\hat{\pi}^e$, the government's best reply is the solution of the following problem:

$$V(b) = \max_{\pi, d, b'} \quad (1-\delta)\left[-(y - k\bar{y})^2 - s(\pi - \pi^\star)^2 - \gamma(b')^2\right] + \delta V(b')$$

$$y = \bar{y} + \theta\left(\pi - \hat{\pi}^e(b)\right) + d,$$

$$b' = d + \frac{(1+r)(1+\hat{\pi}^e(b))b}{1+\pi} - \bar{m}\pi,$$

$$0 \leq b' \leq \bar{b}.$$

Equilibrium Characterization

Back

Proposition

In a Markov Perfect Equilibrium of this dynamic game:

1. No surprise inflation: $\pi^{e\star}(b) = \pi^{\star}(b)$ for all $b \in [0, \bar{b}]$.
2. $V^{\star}$ is a strictly concave and decreasing function of current debt $b \in [0, \bar{b}]$.
3. $\pi^{e\star}$ is an increasing function of $b \in [0, \bar{b}]$.
4. $\pi^{\star}(b)$ is an increasing and differentiable function of $b \in [0, \bar{b}]$.
5. $d^{\star}(b)$ is a decreasing and differentiable function of $b \in [0, \bar{b}]$.
6. Let $s > s'$. Then, $\pi^{\star}(b|s) \leq \pi^{\star}(b|s')$ for all $b \in [0, \bar{b}]$.
7. Let $k > k'$. Then, $\pi^{\star}(b|k) \geq \pi^{\star}(b|k')$ for all $b \in [0, \bar{b}]$.

Dynamic Game: Equilibrium Back

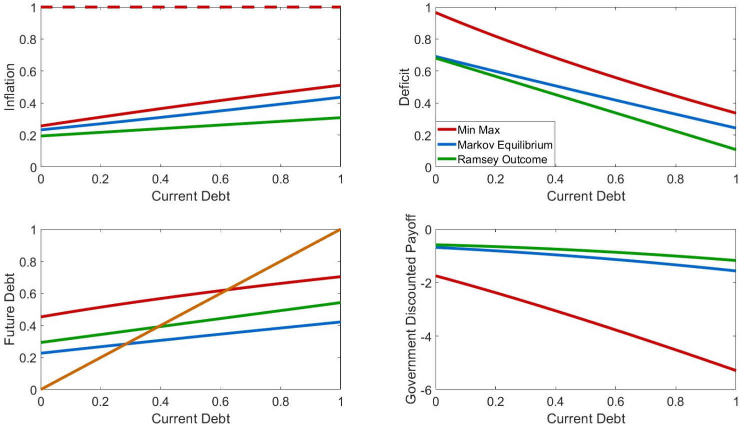
- Given σ_w, the government's (inflation) best reply is π.
- Under convexity and differentiability assumptions on σ_w, the government's best reply is unique.
- Therefore, the government's problem creates a best-reply mapping $\sigma_w \to \pi$.
- Given the utility of wage wetters $-(\pi - \pi^e)^2$, an equilibrium of this game is a fixed point of such mapping.
- Existence of such fixed point is guaranteed by the Schauder Fixed-Point Theorem*.
- Equilibrium characterization can be done using the Implicit Function Theorem, the Envelope Theorem, and the Benveniste-Scheinkman Theorem.

Dynamic Game: Markov Equilibria Back

- Why focus on analyzing Markov equilibria?
- In dynamic games, strategies are more complicated objects than in repeated games, and they live in a "very large" space.
- Non-Markovian "easy" strategies in repeated games, like Grim Trigger, become more complicated to handle, since now strategy has to guarantee that the player does not want to deviate to influence the state transition.
- Also, they do not require agents to coordinate on beliefs about future play, which is a feature of some Non-Markovian strategies as pointed out by Green and Porter (1984).

Markov Equilibria In My Model Back

- In the game analyzed in my paper, the Markov equilibrium yields payoffs that are close to the "first-best", which would be achieved if the government could commit to a policy rule.



Parameter Restrictions

- For the restriction $b' \leq \bar{b}$ to be not binding, I need to impose some parameter restrictions.

Assumption

Let $\bar{y} = 1$. Then, for $b' \leq \bar{b}$ in the stage game for all possible values of $b \in \mathcal{D}$, the parameters must satisfy:

$$r\bar{b} \leq \frac{\bar{b}\gamma(r(s-1)-s)+\gamma k}{\gamma(1+s+\theta)+s}.$$

- All the examples presented in this presentation satisfy this restriction. Back

The Role of γ

- Recall that the flow payoffs for the government are:

$$UG_t = -(y_t - k\bar{y})^2 - s(\pi_t - \bar{\pi})^2 - \gamma b_t^2.$$

- The parameter γ captures the government's aversion to debt.
- Some have suggested to consider $\gamma = 0$ in order to simplify the analysis.
- However, this leads to an uninteresting equilibrium where the government chooses a constant inflation rate.

The Role of γ

Proposition

Suppose $\gamma = 0$ and $\bar{y}(1 + \bar{\pi} - k) \leq r\bar{b}$. Then, in the unique Markov equilibrium of the dynamic game $\pi(b) = \bar{\pi}$, $d(b) = (k-1)\bar{y}$, and $V(b) = 0$ for all $b \in \mathcal{D}$.

The Role of γ

Proof

- Since $\pi^e(b) = \bar{\pi}$, then $y = \bar{y} + \theta(\pi - \bar{\pi}) + d$.
- Then, since the government's flow payoffs are $-(y - k\bar{y})^2 - s(\pi - \bar{\pi})^2$, the government will choose $\pi = \bar{\pi}$ and $d = (k-1)\bar{y}$.
- This gives the government a flow payoff of zero, which is the highest achievable flow payoff.
- In order for the Bellman equation to hold, the value function must be zero.
- Notice that in this case $b' = (k-1)\bar{y} + (1+r)b - \bar{y}\bar{\pi}$, which converges to $b = \max\{0, \frac{\bar{y}(1+\pi-k)}{r}\} \leq \bar{b}$ as long as $\bar{y}(1+\bar{\pi}-k) \leq r\bar{b}$.

- Hence, the term $-\gamma b_t^2$ creates a trade-off between inflation and deficit, which is also impacted by the current debt level. Back

Reputation Framework: Wage Setters Problem Back

- Taking as given (b, ρ) and a conjecture on government behavior $\hat{\pi}^P, \hat{\pi}^I$, wage setters' best reply is characterized by the following problem:

$$\pi^e(b,\rho) = \underset{\pi^e}{\text{argmax}} \;\; \mathbb{E}\left[-\rho(\pi^e - \tilde{\pi}^P(b,\rho))^2 - (1-\rho)(\pi^e - \tilde{\pi}^I(b,\rho))^2\right],$$

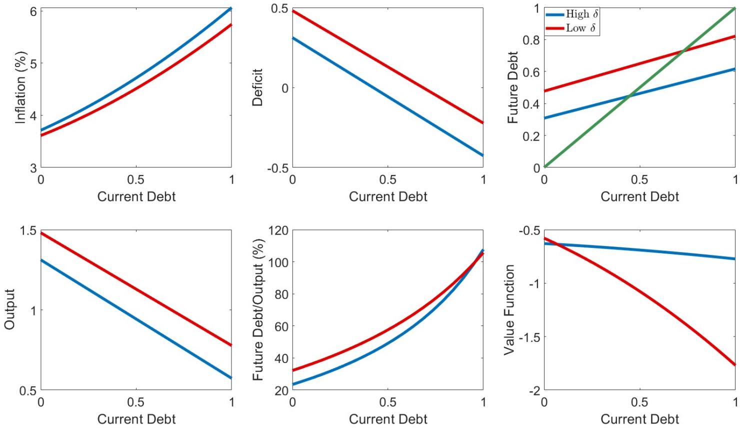
where $\tilde{\pi}^P(b,\rho) = \hat{\pi}^P(b,\rho) + \epsilon_\pi^P$ and $\tilde{\pi}^I(b,\rho) = \hat{\pi}^I(b,\rho) + \epsilon_\pi^I$.

- Then, the best reply of wage setters is given by:

$$\pi^e(b,\rho) = \rho\hat{\pi}^P(b,\rho) + (1-\rho)\hat{\pi}^I(b,\rho).$$

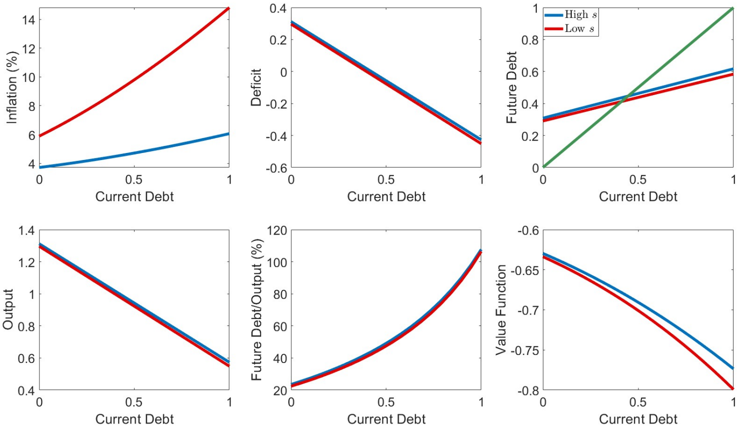
Equilibrium Example: The Role of δ Back

$$V(b) = \max_{\pi, d, b'} \; (1-\delta)\left[-(y-k\bar{y})^2 - s(\pi-\bar{\pi})^2 - \gamma(b')^2\right] + \delta V(b').$$



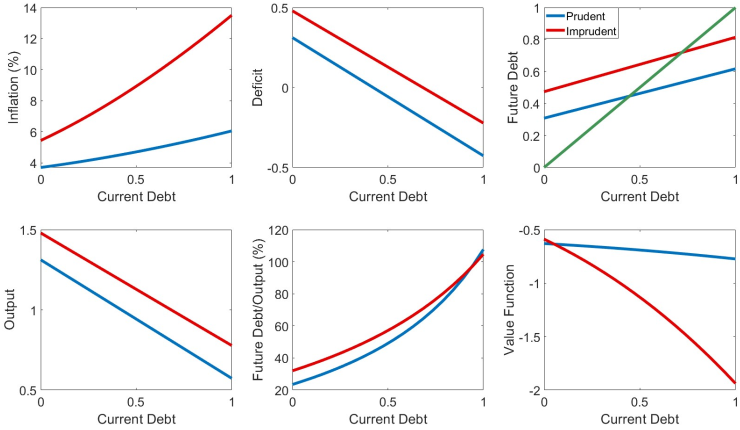
Equilibrium Example: The Role of s Back

$$V(b) = \max_{\pi, d, b'} \ (1-\delta)\left[-(y-k\bar{y})^2 - s(\pi-\bar{\pi})^2 - \gamma(b')^2\right] + \delta V(b').$$



Example: Prudent vs Imprudent Governments Back

$$V(b) = \max_{\pi, d, b'} \; (1-\delta)\left[-(y-k\bar{y})^2 - s(\pi-\bar{\pi})^2 - \gamma(b')^2\right] + \delta V(b').$$



Reputation Framework: Type P's Problem Back

- Taking as given (b, ρ) and a conjecture on wage setters $\hat{\sigma}_w$ as well as the behavior of the government of type I, the government of type P's best reply is characterized by the following problem:

$$V^P(b,\rho) =$$

$$\max_{\pi,d,b'} \quad \mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[(1-\delta)\left[-(\tilde{y}-k\bar{y})^2 - s_P(\tilde{\pi}-\bar{\pi})^2 - \gamma(\tilde{b}')^2\right] + \delta V^P(b',\rho')\right],$$

$$\tilde{y} = \bar{y} + \theta\left(\tilde{\pi} - \hat{\pi}^e(b,\rho)\right) + \tilde{d},$$

$$\tilde{b}' = d + \frac{(1+r)(1+\hat{\pi}^e(b,\rho))b}{1+\tilde{\pi}} - \bar{m}\tilde{\pi},$$

$$\tilde{\pi} = \pi + \epsilon_\pi,$$

$$\tilde{d} = d + \epsilon_d,$$

$$0 \le \tilde{b}' \le \bar{b}.$$

$$\rho' = \frac{\rho g_\pi(\tilde{\pi}-\pi)\, g_d\left(\tilde{d}-d\right)}{\rho g_\pi(\tilde{\pi}-\pi)\, g_d\left(\tilde{d}-d\right) + (1-\rho)g_\pi(\tilde{\pi}-\hat{\pi}^I)\, g_d\left(\tilde{d}-\hat{d}^I\right)}.$$

Reputation Framework: Type I's Problem Back

- Taking as given (b,ρ) and a conjecture on wage setters $\hat{\sigma}_w$, the government of type I's best reply is characterized by the following problem:

$$V^I(b,\rho) = \max_{\pi,d,b'} \quad \mathbb{E}_{\epsilon_\pi,\epsilon_d}\left[-(\tilde{y}-k\bar{y})^2 - s_I(\tilde{\pi}-\bar{\pi})^2 - \gamma(\tilde{b}')^2\right],$$

$$\tilde{y} = \bar{y} + \theta\left(\tilde{\pi} - \hat{\pi}^e(b,\rho)\right) + \tilde{d},$$

$$\tilde{b}' = d + \frac{(1+r)(1+\hat{\pi}^e(b,\rho))b}{1+\tilde{\pi}} - \bar{m}\tilde{\pi},$$

$$\tilde{\pi} = \pi + \epsilon_\pi,$$

$$\tilde{d} = d + \epsilon_d.$$

$$0 \leq \tilde{b}' \leq \bar{b}.$$

Reputation Framework: Updating Rule Back

- Upon observing $(\tilde{\pi}, \tilde{d})$, wage setters will update their beliefs according to Bayes' Rule:

$$\rho'(b, \rho) =$$

$$\frac{\rho g_{\pi}\left(\tilde{\pi} - \pi^{P}(b, \rho)\right) g_{d}\left(\tilde{d} - d^{P}(b, \rho)\right)}{\rho g_{\pi}\left(\tilde{\pi} - \pi^{P}(b, \rho)\right) g_{d}\left(\tilde{d} - d^{P}(b, \rho)\right) + (1 - \rho) g_{\pi}\left(\tilde{\pi} - \pi^{I}(b, \rho)\right) g_{d}\left(\tilde{d} - d^{I}(b, \rho)\right)},$$

Reputation Framework: Equilibrium Proof Back

Proof Theorem

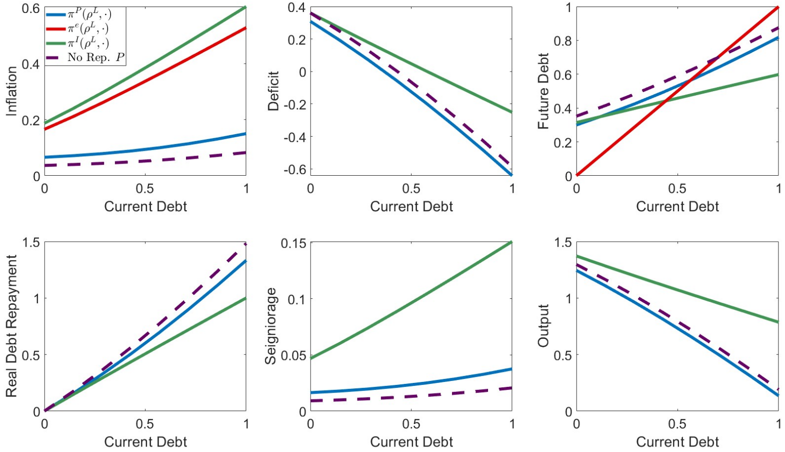
- Let Σ be the set of all functions $\sigma_w : [0, \bar{b}] \times [0, 1] \to \mathbb{R}$ that are strictly convex, uniformly bounded, and have uniformly bounded first derivatives.
- I show that an equilibrium exists by proving that the mapping $\tilde{\pi} : \Sigma_w \to \Omega$ defined by:

$$\tilde{\pi}(\sigma_w)(b, \rho) = \rho\pi^P(b, \rho|\sigma_w) + (1 - \rho)\pi^I(b, \rho|\sigma_w),$$

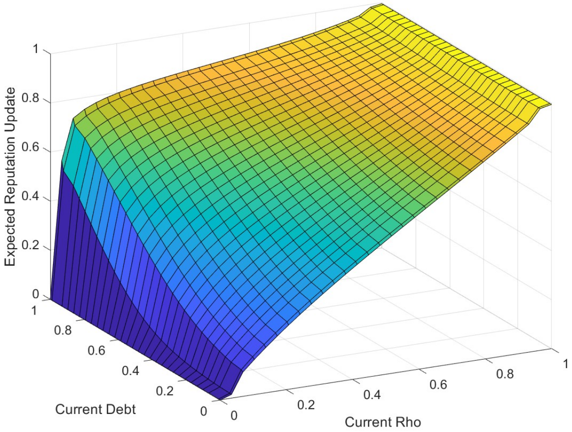
  has a fixed point.
- I consider the Schauder Fixed Point Theorem, and hence I need to show:
  1. Σ_W is a non-empty, convex, and compact subset of a Banach space.
  2. $\tilde{\pi}$ is a continuous mapping with $\Omega \subseteq \Sigma_w$.

Equilibrium: Low Reputation Back

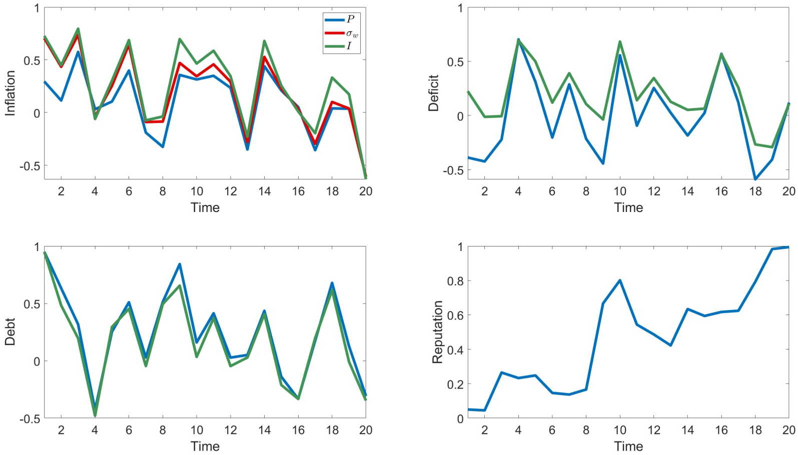
- If the prudent government had no reputation concerns, it would choose lower inflation, which would lead to higher debt, higher real interest rates, and lower seigniorage.



Updating Rule Dynamics Back



Equilibrium: Long-Run Learning Back



Parameter Values for Mexico 1970-2022 Back

Parameter	Interpretation	Value
s_P	Inflation Target Weight Prudent Government	80
s_I	Inflation Target Weight Imprudent Government	5
δ_P	Discount Factor Prudent Government	0.45
δ_I	Discount Factor Imprudent Government	0
ϵ_π	Standard Deviation Inflation Shock	0.15
ϵ_d	Standard Deviation Deficit Shock	0.20
$\bar{y}$	Natural Level of Output	1
θ	Sensitivity of Output to Inflation	0.50
k	Time Inconsistency Parameter	2
γ	Debt Weight	2
$\bar{\pi}$	Inflation Target	3%
r	Interest Rate	5%